## "Precise" Temperament Tuning



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## The Flower of Life/Hexa-Pentakis

Overlapping Squares from the Flower of Life
Forming A Series of Golden Rectangles
(Ratio of 1:1.618 = $\boldsymbol{\Phi}$ ) Across the Flower of Life Structure


Likewise, the Hexa-Pentakis (and Truncated Icosahedron -Archimedean Solid) Bring together the Hexagon and the Golden $\boldsymbol{\Phi}$ ‘Phi-ve’ (Pentagonal) Ratio



Fig.5: Definition of the circular complementary relationship between a set of four numbers with A being one member of the set.

## The Special Role of the Numbers 2 and 3 in the Numbers Series: The Primordial Primes



Fig.4: Prime numbers on the prime moduli adding up to numbers on the central moduli.
"Those numbers that are not prime, while at the same time occupying the prime moduli, are also unique because they are the product of primes larger than or equal to 5 and/or semiprimes only. They are labeled Quasi-prime as to distinguish them from Semi-prime numbers3, which are the product of any two prime numbers, including 2 and 3. "

"There is music in the humming of the strings, there is geometry in the spacing of the spheres."

Mystery of the Tetraktys: $3^{\wedge}$ n and $2^{\wedge}$ n Research


## ‘The Pythagorean Tetraktys’ <br> and Flower of Life



1

2

3

4

## Musical Geometry

The Tetrahedron-Tetraktys Informs the Geometric Relationship between Major and Minor Chords

| 1:1 | Unison |
| :---: | :---: |
| 9:8 | Major 2nd |
| 5:4 | Major 3rd |
| 4:3 | Perfect 4th |
| 3:2 | Perfect 5th |
| 5:3 | Major 6th |
| 9:5 | Major 7th |
| 2:1 | Octave |



1

2

3

4


## The Cuboctahedron

Informs All Major and Minor Chords
"Just" Scale Tuning



## The Major 3rd Problem with ‘Just’ Scale Tuning

-Tuning requires correct mathematical ratios for the Perfect 5th, Major 3rd and Octave Doubling; all other ratios for all other notes can be found

144hz D3: 144hz

"Major 3rd Problem": 843hz $=$ 864hz

D4: 281.25hz vs Major 3rd D4: 288hz
"Major 3rd Problem": 281.25hz $=$ 288hz


## The Ancient Problem of the 'Cube of Delos':

## ^ History

The problem owes its name to a story concerning the citizens of Delos, who consulted the orac e at Delphi in order to learn how to defea a plague sen by Apollo. ${ }^{[5]}$ According to Plutarn , .'] ${ }^{\text {i }}$ it was the citizens of Delos who consulted the oracle at Delphi, seeking a solution for their internal political problems at the time, which had intensified relationships among the citizens. The oracle responded that they must double the size of the altar to Apollo, which was a regular cube. The answer seemed strange to the Delians and they consulted Plato, who was able to interpret the oracle as the mathematical problem of doubling the volume of a given cube, thus explaining the oracle as the advice of Apollo for the citizens of Delos to occupy themselves with the study of geometry and mathematics in order to calm down their passions. ${ }^{[7]}$

## Doubling the cube

From Wikipedia, the free encyclopedia
Doubling the cube, also known as the Delian problem, is an ancient ${ }^{[1]}$ geometric problem. Given the edge of a cube, the problem requires the construction of the edge of a second cube whose volume is double that of the first. As with the related problems of squaring the circle and trisecting the angle, doubling the cube is now known to be impossible using only a compass and straightedge, but even in ancient times solutions were known that employed other tools.
The Egyptians, Indians, and particularly the Greeks ${ }^{[2]}$ were aware of the problem and made many futile
 attempts at solving what they saw as an obstinate but soluble problem. ${ }^{[3][4]}$ However, the nonexistence of a compass-and-straightedge solution was finally proven by Pierre Wantzel in 1837.

In algebraic terms, doubling a unit cube requires the construction of a line segment of length $x$, where $x^{3}=2$; in other words, $x=\sqrt[3]{2}$, the cube root of two. This is because a cube of side length 1 has a volume of $1^{3}=1$, and a cube of twice that volume (a volume of 2 ) has a side length of the cube root of 2 . The impossibility of doubling the cube is therefore equivalent to the statement that $\sqrt[3]{2}$ is not a constructible number. This is a consequence of the fact that the coordinates of a new point constructed by a compass and straightedge are roots of polynomials over the field generated by the coordinates of previous points, of no greater degree than a quadratic. This implies that the degree of the field extension generated by a constructible point must be a power of 2 . The field extension generated by $\sqrt[3]{2}$, however, is of degree 3 .

Volume of the cube doubles


## SOLUTION:



## A Few Unique Properties of 1.26...... $\sqrt[3]{2}$

1.) $1.26^{\wedge} 4=1.26 \times 2 . \ldots . .2 .52$
2.16/1.26 = 1.714285
$=12 / 7$
3.) $\pi(1.26)=1 / .252$
4.) $e / 1.26=2.16$
5.) $e-1 / 1.26=1 / .73$

## ‘Just’ Scale Tuning can be Adjusted Using the Pythagorean Comma Bringing it More in Line with Equal Temperament Tuning

JUST Scale Tuning Requires the
Pythagorean Comma (1.0136)
Adjustment to Fix the
"Major 3rd Problem."
What's the Major 3rd Problem?
$5 / 4$ ( 1.25 x ) is not the correct ratio for the Major 3rd. This is a Fraction, and it should be a CONSTANT of 1.259921 (1.26)

Here's why: The Octave Doubling
Ratio is $2.00 .1\left(1.25^{\wedge} 3\right) \neq 2.00$
In contrast, 1 (1.26^3 = 2.00)

|  | Interval | Ratio to Fundamental Just Scale | Ratio to Fundamental Equal Temperament | $\underline{\text { Ratio }}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $\rightarrow$ Unison | 1.0000 | 1.0000 | +.00 |
|  | Minor Second | $25 / 24=1.0417$ | 1.05946 |  |
|  | Major Second | 9/8 $=1.1250$ | 1.12246 |  |
|  | Minor Third | $6 / 5=1.2000$ | 1.18921 |  |
| E | $\rightarrow$ Major Third | $5 / 4=1.2500$ | 1.25992 | +. 0 |
| ¢ | Fourth | $4 / 3=1.3333$ | 1.33483 |  |
| \% | Diminished Fifth | $45 / 32=1.4063$ | 1.41421 |  |
| 业 | $\rightarrow$ Fifth | $3 / 2=1.5000$ | 1.49831 | -. 001 |
| $\stackrel{\rightharpoonup}{\mathbf{o}}$ | Minor Sixth | $8 / 5=1.6000$ | 1.58740 |  |
|  | Major Sixth | $5 / 3=1.6667$ | 1.68179 |  |
|  | Minor Seventh | $9 / 5=1.8000$ | 1.78180 |  |
|  | Major Seventh | $15 / 8=1.8750$ | 1.88775 |  |
|  | $\rightarrow$ Octave | 2.0000 | 2.0000 | +. 00 |

$$
\begin{aligned}
& \text { "The ratio of } 5 / 4 \text { (1.25) is wholly inadequate as a viable approach for the } \\
& \text { Major 3rd, as, if continued, will never achieve a correct doubling of an } \\
& \text { octave. This is the interval that totally destroys "Just" Tuning as a viable } \\
& \text { uning approach. It is so obvious in fact, that I believe that Pythagoras must }
\end{aligned} \begin{gathered}
\text { R. Grant } \\
7-17-20
\end{gathered}
$$

have intentionally obfuscated it to conceal the correct 1.26 ratio."

But, Does Nature Make Such Linear Separations for Musical Notes?
How to reconcile the 'convenience' of Equal Temperament with the clean mathematical intervals of 'Just' Scale Tuning?

## ‘The Controversy’

"Just intonation emphasizes bright, booming perfect thirds, but the way the maths works out, that means the fifth between $D$ and $A$ is pushed out of tune. Equal temperament pretends you can have it both ways; Just Intonation makes a conscious choice about which intervals matter most. The argument goes that equal temperament is becoming increasingly streamlined and corporate, and man's capacity to hear and feel subtle inflections of tuning is now in slow retreat."
-Philip Clark

## Equal Temperament

12 Equal/Linear Separations of the Frequency Range of an Octave

## Equal Temperament Transition From Just Tuning

| Interval | Ratio to Fundamental <br> Just Scale | Ratio to Fundamental <br> Equal Temperament |
| :---: | :---: | :---: |
| Unison | 1.0000 | 1.0000 |
| Minor Second | $25 / 24=1.0417$ | 1.05946 |
| Major Second | $9 / 8=1.1250$ | 1.12246 |
| Minor Third | $6 / 5=1.2000$ | 1.18921 |
| Major Third | $5 / 4=1.2500$ | 1.25992 |
| Fourth | $4 / 3=1.3333$ | 1.33483 |
| Diminished Fifth | $45 / 32=1.4063$ | 1.41421 |
| Fifth | $3 / 2=1.5000$ | 1.49831 |
| Minor Sixth | $8 / 5=1.6000$ | 1.58740 |
| Major Sixth | $5 / 3=1.6667$ | 1.68179 |
| Minor Seventh | $9 / 5=1.8000$ | 1.78180 |
| Major Seventh | $15 / 8=1.8750$ | 1.88775 |
| Octave | 2.0000 | 2.0000 |

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| Equal Temperament Equation |  | 'Reduced |
| :---: | :---: | :---: |
| 1.00 | 1.00 | 1.00 |
| $\sqrt[12 / 1]{2}$ | 1.059463094359295 | $\sqrt[12]{2}$ |
| 12/2/2 | 1.122462048309373 | $6_{2}$ |
| $\sqrt[12 / 3]{ } 2$ | 1.189207115002721 | $\sqrt[4]{2}$ |
| $\sqrt[12 / 4]{2}$ | 1.259921049894873 | $\sqrt[3]{2}$ |
| $\sqrt[12 / 5]{2}$ | 1.334839854170034 | ${ }^{12 / 5} / 2$ |
| ${ }^{12 / 6} 2$ | 1.414213562373095 | $\sqrt{ } 2$ |
| $\sqrt[{12 / \sqrt{2}}]{ }$ | 1.498307076876681 |  |
| $\sqrt[12 / 8]{2}$ | 1.587401051968199 | $\sqrt[3 / 2]{2}$ |
| ${ }^{12 / 9} 2$ | 1.681792830507429 | $\sqrt[4 / 3]{ } 2$ |
| $\sqrt[12 / 10]{2}$ | 1.781797436280679 | $\sqrt{6 / 5} 2$ |
| $\sqrt[12 / 1]{1} 2$ | 1.887748625363387 | ${ }^{12 / 111} / 2$ |
| 2.00 | 2.00 | 2.00 |

## The Mathematics of Equal Temperament is Based Upon $\sqrt{ }$ 2...

With One Adjustment to the Major 3rd (from 1.25x to 1.26x) 'Just' Scale Tuning Reconciles with Equal Temperament in a New Tuning:'Precise’ Temperament Tuning in 432hz

| Interval | Ratio to Fundamental <br> Just Scale | Ratio to Fundamental <br> Equal Temperament |
| :---: | :---: | :---: |
| Unison | 1.0000 | 1.0000 |
| Minor Second | $25 / 24=1.0417$ | 1.05946 |
| Major Second | $9 / 8=1.1250$ | 1.12246 |
| Minor Third | $6 / 5=1.2000$ | 1.18921 |
| Major Third | $5 / 4=1.2500$ | 1.25992 |
| Fourth | $4 / 3:=1.3333$ | 1.33483 |
| Diminished Fifth | $45 / 32=1.4063$ | 1.41421 |
| Fifth | $3 / 2:=1.5000$ | 1.49831 |
| Minor Sixth | $8 / 5=1.6000$ | 1.58740 |
| Major Sixth | $5 / 3=1.6667$ | 1.68179 |
| Minor Seventh | $9 / 5=1.8000$ | 1.78180 |
| Major Seventh | $15 / 8=1.8750$ | 1.88775 |
| Octave | 2.0000 | 2.0000 |

Ratio to
Fundamental
Precise Temper
1.00
1.058
1.125
1.190
1.26
1.333
1.414
1.5
1.587
1.886
1.089
R. Grant
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## 'Precise' Temperament Tuning in 432hz

| 'Precise Temperament' vs Equal/Just |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  | Precise Tempered$\underline{\text { 432hz }}$ | Ratio to <br> Fundamental <br> Precise Temp. |
| Interval | Ratio to Fundamental Just Scale | Ratio to Fundamental Equal Temperament | Equal Tempered$432 \mathrm{hz}$ |  |  | $\underline{\text { Ratio }}$ |  |  |
| Unison | 1.0000 | 1.0000 | 432hz | +.081 hz | A5 | +. 000019 | 432.081216 hz | 1.00 |
| Minor Second | $25 / 24=1.0417$ | 1.05946 | 457.688 hz | -.459hz | A\# | -. 001 | 457.2288 hz | 1.058 |
| Major Second | $9 / 8=1.1250$ | 1.12246 | 484.903hz | +1.188hz | B | +. 00245 | 486.091368 hz | 1.125 |
| Minor Third | $6 / 5=1.2000$ | 1.18921 | 513.737 hz | +.742hz | c | +. 0014 | 514.4791038912 hz | 1.190 |
| Major Third | $5 / 4=1.2500$ | 1.25992 | 544.285 hz | +.137hz | C\# | +. 00025 | 544.42233216 hz | 1.26 |
| Fourth | $4 / 3=1.3333$ | 1.33483 | 576.650hz | -.541 hz | D | -. 00094 | 576.108288hz | 1.333 |
| Diminished Fifth | $45 / 32=1.4063$ | 1.41421 | 610.940 hz | . 000 hz | D\# | .000hz | 610.9402589451771 hz | z 1.414 |
| Fifth | $3 / 2=1.5000$ | 1.49831 | 647.268 hz | +.975hz | E | +. 0015 | 648.243670902912 hz | 1.50 |
| Minor Sixth | $8 / 5=1.6000$ | 1.58740 | 685.757 hz | +.215hz | F | +. 00031 | 685.9721385216 hz | 1.587 |
| Major Sixth | $5 / 3=1.6667$ | 1.68179 | 726.534 hz | -.774hz | F\# | -. 001 | 72.76 hz | 1.68 |
| Minor Seventh | $9 / 5=1.8000$ | 1.78180 | 769.736 hz | +1.982hz | G | +. 0025 | 771.7186558368 hz | 1.786 |
| Major Seventh | $15 / 8=1.8750$ | 1.88775 | 815.507 hz | +1.126hz | G\# | +. 0014 | 816.63349824hz | 1.889 |
| Octave | 2.0000 | 2.0000 | 864hz | +.162hz | A6 | +. 00018 | 864.162432hz | 2.00 |
|  |  |  |  |  |  |  |  | $\underset{\substack{\text { R. Grant } \\ \hline \\ \hline}}{ }$ |

## "Just" Scale Tuning

| 1:1 | Unison |
| :---: | :---: |
| 9:8 | Major 2nd |
| 5:4 | Major 3rd |
| 4:3 | Perfect 4th |
| 3:2 | Perfect 5th |
| 5:3 | Major 6th |
| 9:5 | Major 7th |
| 2:1 | Octave |



## "Equal" Temperament Tuning

| $\xrightarrow{\text { 1:1 }}$9:8 Major 2nd <br> 5:4 Major 3rd |  |
| :--- | :--- |
| 4:3 | Perfect 4th |
| 3:2 | Perfect 5th |
| 5:3 | Major 6th |
| 9:5 | Major 7th |
|  |  |
| 2:1 | Octave |



## "Precise" Temperament Tuning

| 1:1 | Unison |
| :---: | :---: |
| 9:8 | Major 2nd |
| 5:4 | Major 3rd |
| 4:3 | Perfect 4th |
| 3:2 | Perfect 5th |
| 5:3 | Major 6th |
| 9:5 | Major 7th |
| 2:1 | Octave |




Mathematical Interval Perfection of Major 2nd, Perfect 5th and Perfect 4th

Not versatile across Key Transitions
"Equal" Temperament Tuning


Why Precise Temperament Tuning?


## 'Precise Temperament'

Musical Geometry
The Cuboctahedral Structure Informs Major and Minor Chords


## 'Precise Temperament'

Musical Geometry
The Cuboctahedral Structure Informs Major and Minor Chords


But What About those Very Unique Decimal Extensions that Appear Using 1.26 as the Major 3rd Interval in Precise Temperament Tuning?......

'Just Tuning' Intervals as Fractals in a Blockchain-like Configuration of Chords?

Let's take a close look at a few of these.....

D3: 144.027072hz


A1: 27 hz
D2: 72hz
$A$ is the Major 5th of $D$

B4: 243.045684hz


F\#1: 45hz
F4: 684hz
Perfect 4th and Major 7th of F\#

G\#6: 914.4576hz


D3 144hz
D5: 576hz
D is Perfect 4th; G\# is the Major 7th of A

A5: 432.081216hz


E2: 81 hz
$E$ is the Perfect 5th of $A$

D5: 576.108288hz


A3: 108hz
D4: 288 hz
D is the Major 4th of A

# Blah, blah, blah...... but how does it sound? 

https://soundcloud.com/jasonmartineau/tracks


How does 'Precise' Temperament Tuning effect the inherent Undertone Series? What is the potential for new sound-based technologies? How might this advance our understanding of gravity, radiation, time and energy?

More research and time will tell......
~Music of the Stan Tetrahedron ~


## The Flower of Life <br> Squares in Rotational Positions



## The Flower of Life

Squares Only


## The Flower of Life

Squares Only


## The Flower of Life

Squares in Rotation
$120^{\circ}$ Rotation


## The Flower of Life

Circles Only


## The Flower of Life

Circles and Squares


## The Flower of Life

Squares Only


Geometry and Music: One and the Same.....

