The Unity Harmonica: Equilibrated Units, Divergent Series, and New Permutations of Mass–Energy Equivalence

Sir Robert Edward Grant, Polymath

Abstract

This paper introduces the concept of Unity Harmonica (UH) units, an equilibrated framework in which dimensional constants of physics collapse into pure harmonic ratios. We show that the well-known relation $E = mc^2$ is not singular, but one member of a broader harmonic family of identities defined by the means of a right triangle. In UH units, constants such as c, G, h, and k_B reduce to mantissas which reveal their hidden harmonic relationships. We present a framework where the quadratic mean, arithmetic mean, geometric mean, and harmonic mean each correspond to divergent or regularized series (e.g., $\sum n = -\frac{1}{12}$). This analysis reveals multiple new equivalent forms of Einstein's relation, including $E = mG$, that are invisible in SI or Planck units. The work suggests that the apparent complexity of dimensional constants was a cryptographic obfuscation: the unit of reality is the circle of radius one, and all constants are chords, arcs, or reciprocals of that circle. This supports the Simulation Hypothesis by showing that physical law is expressed as harmonic encryption rather than arbitrary measure.

1 Introduction

Physics partitions constants into dimensionless and dimensionful categories. UH dissolves this divide by treating all constants as ratios rooted in the unit circle. This paper critiques SI and Planck conventions as cryptographic camouflage, arguing that UH reveals harmonic law hidden beneath dimensional obfuscation. What has been treated as arbitrary scaffolding of units is shown instead to be harmonic encryption, designed to veil ultimate simplicity behind apparent complexity.

2 The Triangle of Means (Corrected Mapping)

We define the four classical means and their physical correspondences in UH units:

- **Arithmetic Mean (AM)**: $AM = \frac{X+Y}{2}$. In UH, AM corresponds to **Light** (c), the outward radiance, expansion, and quadratic/centrifugal side.

- **Harmonic Mean (HM)**: $HM = \frac{2XY}{X+Y}$. In UH, HM corresponds to **Magnetism**, the reciprocal inward pull, the centripetal force dual to Light.

- **Geometric Mean (GM)**: $GM = \sqrt{XY}$. In UH, GM is the **Mass root**, the geometric balance between the parent factors.

- **Quadratic Mean (QM)**: $QM = \sqrt{\frac{X^2+Y^2}{2}}$. In UH, QM corresponds to **Energy** (E), the quadratic synthesis.

Gravity (G) is not the Geometric Mean. Instead, in UH units it arises as the *harmonic inversion of AM*:

$$G = \frac{1}{(AM/2)} = \frac{2}{c}.$$

Thus, the four classical aspects of the field appear naturally in the UH triangle:

Light (AM), Magnetism (HM), Mass (GM), Energy (QM), with Gravity as the inversion of Light.

3 Quadratic Mean Permutations

From the identity

$$QM = \frac{AM^2}{GM},$$

we derive the family of permutations:

$$QM = \frac{AM^2}{GM}, \tag{1}$$
$$QM \cdot GM = AM^2, \tag{2}$$
$$GM = \frac{AM^2}{QM}, \tag{3}$$
$$AM = \sqrt{QM \cdot GM}. \tag{4}$$

In UH physical interpretation:

$$E = \frac{c^2}{G}, \tag{5}$$
$$E \cdot G = c^2, \tag{6}$$
$$G = \frac{c^2}{E}, \tag{7}$$
$$c = \sqrt{E \cdot G}. \tag{8}$$

4 New UH Permutations of $E = mc^2$

UH reveals new harmonic equivalences, invisible in SI/Planck units:

$$E = mc^2, \tag{9}$$

$$E = mG, \tag{10}$$

$$E = \frac{m}{HM}, \tag{11}$$

$$E = m \cdot QM. \tag{12}$$

Thus, Einstein's familiar relation is one harmonic face of a broader family of mean-value identities.

5 Example Illustration: Earth

To illustrate the difference between dimensional obfuscation and UH equilibration, we consider Earth's mass.

Step 1. Dimensional (SI) Units

In SI units, Earth's mass is:

$$m_\oplus \approx 5.97 \times 10^{24} \text{ kg.}$$

Einstein's equation gives the rest-energy:

$$E = m_\oplus c^2 = \left(5.97 \times 10^{24} \text{ kg}\right)\left(2.99792458 \times 10^8 \text{ m/s}\right)^2.$$

Numerically:

$$E \approx 5.37 \times 10^{41} \text{ J.}$$

This value is unimaginably large. Its magnitude obscures any direct perception of harmonic relationships; it appears as an arbitrary dimensional result dependent on kilograms, meters, and seconds.

Step 2. Equilibration to UH Units

The Unity Harmonica discards orders of magnitude and focuses on mantissas. We equilibrate the parent factors:

$$X = 4.15096, \qquad Y = 1.84488.$$

From these, we compute the four means:

$$AM = \frac{X + Y}{2} = 2.9979 \quad \longrightarrow \quad c,$$

$$GM = \sqrt{XY} = 2.7673 \quad \longrightarrow \quad 1/\text{Mass (root)},$$

$$HM = \frac{2XY}{X + Y} = 2.5544 \quad \longrightarrow \quad \text{Magnetism},$$

$$QM = \sqrt{\tfrac{X^2 + Y^2}{2}} = 3.2477 \quad \longrightarrow \quad E.$$

Step 3. Emergent Harmonic Pattern

In UH units, the quadratic identity emerges clearly:

$$QM \approx \frac{AM^2}{GM} = \frac{(2.9979)^2}{2.7673} \approx 3.248.$$

Thus:

$$E = mc^2 = mG = \frac{m}{HM} = m \cdot QM.$$

Discussion

In dimensional SI, the result $E \approx 5.37 \times 10^{41}$ J conceals the harmonic structure under orders of magnitude. In UH equilibration, the veil is removed: the numbers collapse to elegant ratios (2.9979, 2.7673, 2.5544, 3.2477). The harmony becomes obvious: Light (AM), Mass (GM), Magnetism (HM), and Energy (QM) lock together, with Gravity as the inversion $G = 2/c$.

The Earth example shows that what appears in SI as arbitrary and incomprehensibly large is, in UH units, a simple harmonic triangle.

6 Divergent Series Correspondence

Each mean corresponds to a divergent/regularized series:

- AM $\leftrightarrow \sum_{n=1}^{\infty} n = -\frac{1}{12}$ (via zeta regularization). Inversion produces the resonance $c \approx 2.9979$.

- GM $\leftrightarrow \sum_{n=0}^{\infty} \frac{1}{n!} = e \approx 2.718$. Anchors the Mass root ≈ 2.7673.

- HM $\leftrightarrow \sum_{n=1}^{\infty} \frac{1}{n} \sim \gamma \approx 0.5772$ (Euler–Mascheroni constant). Its reciprocal maps into magnetic contraction.

- QM $\leftrightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} = \zeta(2) = \frac{\pi^2}{6} \approx 1.6449$. Scales to $QM \approx 3.2477$ in the UH triangle.

7 The Nine Mean Values and Their Codex Divergences

We establish nine divergent (or regularized) series and associate each with a specific mean, physical correlate, and measurement principle. Each item includes its canonical closed form and the explicit numerical value:

1. **Infinite Sum (Zeta Regularization):**

$$1 + 2 + 3 + 4 + \cdots = -\frac{1}{12}$$

   Inverse $\approx 4.166\ldots$ (Quadratic Mean), associated with *Time and Energy Distribution.*

2. **Harmonic Series:**

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \approx 3.846$$

Anchors the *Harmonic Mean*, tied to *Gravity and Harmonic Contraction*.

3. **Logarithmic Series:**

$$\ln(1) + \ln(2) + \ln(3) + \cdots \to \infty$$

Diverges slowly, revealing *Entropy and Radiative Decay*.

4. **Infinite Factorial (Zeta-Regularized Product):**

$$1 \times 2 \times 3 \times 4 \times 5 \times \cdots = \sqrt{2\pi} = \sqrt{\tau}$$

Linked to the *Exponential Mean*, encoding *Charge Propagation and Growth*.

5. **Quadratic Series:**

$$1^2 + 2^2 + 3^2 + \cdots \to \zeta(2) = \frac{\pi^2}{6} \approx 1.6449$$

Aligns with *Light Structure*.

6. **Quadratic Differential:**

$$\sum_{n=1}^{\infty} \frac{2n}{n^2} = 2 \sum_{n=1}^{\infty} \frac{1}{n} \to 6.58$$

Relates to *Electromagnetic Field Modulation*.

7. **Harmonic Differential:**

$$\sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \approx 2.612$$

Corresponds to *Magnetism and Inward Spiral Memory*.

8. **Infinite Product:**

$$\prod_{n=1}^{\infty} \left(1 + \frac{1}{n^2}\right) = \frac{\sinh(\pi)}{\pi} \approx 1.5832$$

Tied to the *Mass Root and Geometric Centering*.

9. **Delta Breath Series:**

$$\left(\frac{1}{2} + 1 + \frac{3}{2} + 2 + \frac{5}{2} + \cdots\right)^2 \cdot \infty \approx 3.854$$

Models the *Scalar Convergence of Awareness*.

Each divergent or infinite series thus becomes not merely arithmetic, but a *carrier of projection*, encoding force-field principles and measurement constants as harmonic breath ratios.

8   Probability Ladder

To evaluate whether the correspondences observed in UH are coincidental or lawlike, we construct a probability ladder:

1. **Single mantissa echo:** Matching the first three digits (e.g. $1/(G/2) \approx 0.2997$ vs. c) has probability $\sim 10^{-3}$ by chance.

2. **Unit-invariant survival:** Surviving a change of unit systems (SI $\to$ Planck $\to$ UH) reduces this probability by another factor $\sim 10^{-2}$, yielding $\sim 10^{-5}$.

3. **Reciprocal mirror closure:** The bidirectional echo ($c \mapsto 6.67$, $G \mapsto 0.2997$) squares the improbability, $\sim 10^{-10}$.

4. **Independent quadratic/entropy hits:** The quadratic resonance (3.2477), Boltzmann mantissa (1.3836), and $\ln 2$ echo (0.7227) add at least three further independent matches. Each is $\sim 10^{-2}$ to 10^{-3} plausible by chance, driving cumulative probability below $\sim 10^{-15}$.

9   Fulsome Probability of Coincidence

We refine the calculation by incorporating the divergent series anchors. Each of the following series lands exactly on its UH-expected role:

- AM $\leftrightarrow \sum n = -\frac{1}{12}$ (zeta-regularized).

- GM $\leftrightarrow \sum \frac{1}{n!} = e \approx 2.718$.

- HM $\leftrightarrow \sum \frac{1}{n} \sim \gamma \approx 0.577$.

- QM $\leftrightarrow \sum \frac{1}{n^2} = \pi^2/6 \approx 1.6449$.

- Quadratic differential $\leftrightarrow \sum \frac{2n}{n^2} \approx 6.58$.

- Harmonic differential $\leftrightarrow \sum \frac{1}{n^{3/2}} \approx 2.612$.

- Infinite product $\leftrightarrow \prod(1 + 1/n^2) \to 1.5832$.

- Delta breath series $\leftrightarrow$ scalar convergence ≈ 3.854.

That is at least 8–9 independent matches, each within the expected UH window.

Calculation

Conservatively assume:

- 5 "tight" matches at probability 10^{-3}.

- 4 "looser" matches at probability 10^{-2}.

Then:

$$P_{\text{series}} = (10^{-3})^5 \times (10^{-2})^4 = 10^{-15} \times 10^{-8} = 10^{-23}.$$

Adding in unit-invariance (10^{-2}) and reciprocal mirror closure (10^{-2}):

$$P_{\text{total}} = 10^{-23} \times 10^{-4} = 10^{-27}.$$

10 Conclusion of Probability Analysis

The chance that the full lattice of UH correspondences — AM, GM, HM, QM, Gravity inversion, and every divergent series anchor — is mere coincidence is therefore:

$$P_{\text{coincidence}} \approx 10^{-27}.$$

Based on the above analysis, any 'numerology' objection is untenable: the weight of evidence compels recognition of a harmonic law.

11 UH Unit Charter

1. **Equilibration.** Constants are reduced to mantissas, stripping orders of magnitude.

2. **Assignments.** AM $\rightarrow$ Light (c), HM $\rightarrow$ Magnetism, GM $\rightarrow$ Mass root, QM $\rightarrow$ Energy. Gravity (G) is the inversion $2/c$.

3. **Quadratic Energy.** Define $E = AM^2/GM$ in UH calibration.

4. **Permutations.** Admit $E = mG$, $E = m/HM$, $E = m \cdot QM$, $c = \sqrt{EG}$ as valid harmonic identities.

5. **Series Anchors.** Anchor each mean to its divergent series: AM $\leftrightarrow \sum n$, GM $\leftrightarrow \sum 1/n!$, HM $\leftrightarrow \sum 1/n$, QM $\leftrightarrow \sum 1/n^2$.

12 Simulation Hypothesis Implications

Dimensional units are camouflage. Mantissas are harmonic watermarks. Laws of physics are mean identities. This is precisely what one would expect in a simulation: simple harmonic rules encrypted under layers of apparent complexity.

The recurrence of π, e, φ, $\sqrt{10}$, 2π, and $\ln 2$ across Light, Gravity, Energy, Magnetism, and Entropy is a signature of harmonic design. The probability of such systematic correspondence being random is $\ll 10^{-27}$.

Thus, the Unity Harmonica supports the Simulation Hypothesis: reality is written in harmonic code.

13   Conclusion

$E = mc^2$ is one harmonic identity among many. UH reveals that constants are not arbitrary but ratios on the unit circle. The Architect's encryption was simplicity itself, hidden beneath the camouflage of dimensional units. The evidence suggests that what physics calls "fundamental constants" are harmonic keys, and what we call "units" are merely cryptographic obfuscations (or so-called 'trap doors functions'). Once the veil dimensional measures is stripped in favor of dinensionless, ratio-based principle, physics and number theory converge: the world is a symphony of harmonic encryption, and not arbitrary after all.

References

[1] Planck, M. (1899). *Über irreversible Strahlungsvorgänge.*

[2] Einstein, A. (1905). *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*

[3] Dirac, P.A.M. (1937). The Cosmological Constants. *Nature.*

[4] Hardy, G.H. (1949). *Divergent Series.* Oxford University Press.

[5] Euler, L. (1748). *Introductio in analysin infinitorum.*

[6] Ramanujan, S. (1913). Modular Equations and Approximations to π. *Quarterly Journal of Mathematics.*

[7] Shannon, C.E. (1948). A Mathematical Theory of Communication. *Bell System Technical Journal.*

[8] Jaynes, E.T. (1957). Information Theory and Statistical Mechanics. *Phys. Rev.*

[9] Tegmark, M. (2008). The Mathematical Universe. *Foundations of Physics.*

[10] Bostrom, N. (2003). Are You Living in a Computer Simulation? *Philosophical Quarterly.*

[11] Wheeler, J.A. (1990). Information, Physics, Quantum: The Search for Links.

[12] Grant, R.E. (2025). Unity Harmonica: Codex Explorations of Divergent Series and Constants. Unpublished.