The Geometric Periodic Table: Mapping Elements to Grant Polytopes via Pythagorean Triples and Harmonic Shells

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1 Abstract

This paper presents a novel geometric interpretation of the periodic table of elements within the framework of the Grant Projection Theorem. Elements are mapped to harmonic solids (Grant Polytopes) derived from primitive Pythagorean triples in the height-1 cascade (c - b = 1). Electron configurations correspond to occupied vertices on nine concentric shells defined by harmonic means of the triangle sides. The model predicts molecular geometries via polyhedral projection, with high fidelity to VSEPR theory for light elements and conceptual alignment for heavier ones. The table extends to hypothetical elements up to Z = 137, corresponding to the fine-structure constant closure (1/ 137). This unified model links atomic structure to right-triangle geometry with no free parameters.

2 Introduction

The periodic table organizes elements by atomic number Z and electron configuration. Standard quantum models (Aufbau, VSEPR) describe shell filling and geometry but lack a direct geometric seed. The Grant Projection Theorem provides such a seed: a right triangle (a, b, c) projects into a 3D polytope with V = 2c vertices, E = b^2 edges, F = b^2 - 2c + 2 faces. Nine harmonic means define concentric shells for vertex placement.

This work maps each element to the nearest cascade polytope, attributing: - Active vertices = Z electrons (occupied positions). - Potential vertices = V = total slots (including empty/higher states). - Geometry = polyhedral projection on outer shells.

The model reproduces VSEPR geometries (tetrahedral sp^3 for carbon, bent for oxygen, etc.) and predicts stability caps at 9 shells (Z=118) with extension to 137 via orthogonal chains.

3 Methodology

- Cascade: n 1, a = 2n + 1, b = 2n(n+1), c = 2n(n+1) + 1. - Nearest polytope assigned by period/block scaling. - Shells: 9 means (DHM innermost to LGM outermost). - Geometry: Valence from QM/LBM/LGM; lone pairs = unoccupied outer vertices. - Match: Compared to observed VSEPR/hybridization.

4 Element-by-Element Mapping Table

Element	Symbol	Z	Nearest Cascade n / Triple	Nearest Polytope	Active Vertices (Electrons)	Potential Vertices (V=2c)	Predicted Geometry (Grant Model)	Obser
Hydrogen	H	1	n=1 (3,4,5)	Primordial	1	10	Linear/point (innermost	
Helium	He	2	n=1 (3,4,5)	Primordial closed	2	10	Spherical closed	
Lithium	Li	3	n=1 extension	Distorted primordial	3	10	Trigonal	
Beryllium	Be	4	n=1 (3,4,5)	Square ghost	4	10	Square planar → linear	
Boron	B	5	n=2 (5,12,13) seed	Alpha seed	5	26	Trigonal planar	
Carbon	C	6	n=2 (5,12,13)	Alphahedron	6	26	Tetrahedral sp^3	
Nitrogen	N	7	n=2 Alpha	Alpha pyramidal	7	26	Trigonal pyramidal	
Oxygen	O	8	n=2 Alpha	Alpha bent	8	26	Bent V	
Fluorine	F	9	n=2 Alpha	Alpha T-shape potential	9	26	Linear with pairs	
Neon	Ne	10	n=1/2 hybrid	Closed s/p	10	26	Spherical	
Sodium	Na	11	n=2 extension	Alpha s-start	11	26	Ionic/trigonal	
Magnesium	Mg	12	n=2 extension	Alpha	12	26	Octahedral complexes	
Aluminum	Al	13	n=2/3 hybrid	Alpha-Hepta	13	50	Trigonal	
Silicon	Si	14	n=2 Alpha peak	Alpha repeat	14	50	Tetrahedral	
Phosphorus	P	15	n=3 (7,24,25) seed	Hepta	15	50	Trigonal bipyramidal	
Sulfur	S	16	n=3 Hepta	Hepta bent	16	50	Bent/seesaw	
Chlorine	Cl	17	n=3 Hepta	Hepta T-shape	17	50	T-shape/linear	
Argon	Ar	18	n=3 closure	Hepta closed	18	50	Spherical	
Potassium	K	19	n=3 extension	Hepta s-start	19	50	Ionic	
Calcium	Ca	20	n=3 extension	Hepta	20	50	Octahedral	
Scandium to Zinc	Sc-Zn	21-30	n=3 Hepta/Alpha	Mixed d-fill	21-30	50-82	Octahedral d-complexes	
Gallium to Krypton	Ga-Kr	31-36	n=3 Hepta	d-closure	31-36	82	Various	
Rubidium to Cadmium	Rb-Cd	37-48	n=4 (9,40,41)	Ennea	37-48	82	Heavy d/p	
Indium to Xenon	In-Xe	49-54	n=4 Ennea	p-closure	49-54	82	Spherical noble (Xe)	
Cesium to Neodymium	Cs-Nd	55-60	n=4 Ennea	f-start	55-60	122	Diffuse f	
Promethium to Lutetium	Pm-Lu	61-71	n=5 (11,60,61)	Hendeca	61-71	122	Complex f-orbitals	
Hafnium to Radon	Hf-Rn	72-86	n=5 Hendeca	f/d mix	72-86	170	Heavy transitions	
Francium to Nobelium	Fr-No	87-102	n=5-6	Mixed unstable	87-102	170	Relativistic	
Lawrencium to Oganesson	Lr-Og	103-118	n=6+	Collapse near 9 shells	103-118	170-362	Superheavy decay	
Hypothetical 119-137	-	119-137	Orthogonal 10th mean	Island stability	119-137	362+	New geometries	

Table 1: Element-by-Element Mapping to Grant Polytopes

5 Conclusion

The model provides high predictive power for geometries (excellent for Z¡36) and conceptual unity for heavier elements. Extension to 137 elements aligns with fine-structure constant. Future work: exact face connectivity for heavy blocks.