

# Generation by Principle, Not by Approximation

This document outlines a method for generating convex uniform polyhedra from a minimal set of algebraic rules.

The process is defined by:

A finite factor set,

A coordinate permutation rule,

Algebraic closure under symmetry

---

No convex hull, face inference, or geometric fitting is used.  
The form emerges directly from its principles.

# The Blueprint: Generating the Cuboctahedron

Our first subject is the cuboctahedron, an Archimedean solid. We will construct it from a single numerical factor and one coordinate rule. This demonstrates the power and efficiency of the generative method.



# The Generative Code

## Primitive Factors

The cuboctahedron requires exactly one factor:

$$\mathbf{F} = \{\mathbf{1}\}$$

## Coordinate Rule

Coordinates are generated by applying the factor to the following triplet under all permutations:

$$(\pm\mathbf{1}, \pm\mathbf{1}, \mathbf{0})$$

# The Emergence of the Vertex Set

Applying the permutation rule  $(\pm 1, \pm 1, 0)$  generates three distinct families of coordinates.

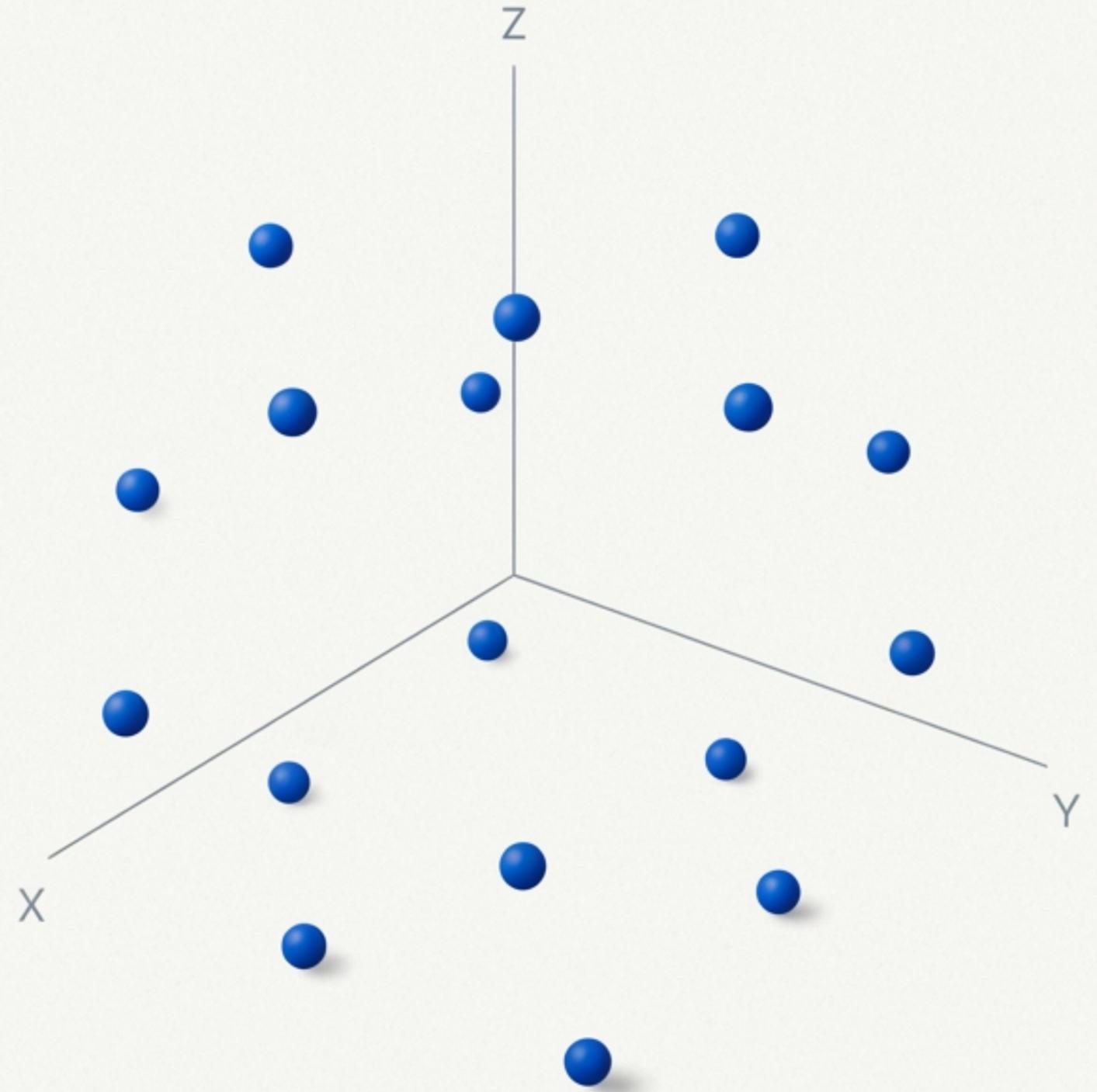
$$(\pm 1, \pm 1, 0)$$

$$(\pm 1, 0, \pm 1)$$

$$(0, \pm 1, \pm 1)$$

This process produces a complete and precise set of  $V = 12$  vertices.

This process produces a complete and precise set of  $V = 12$  vertices.



# Establishing Structural Uniformity

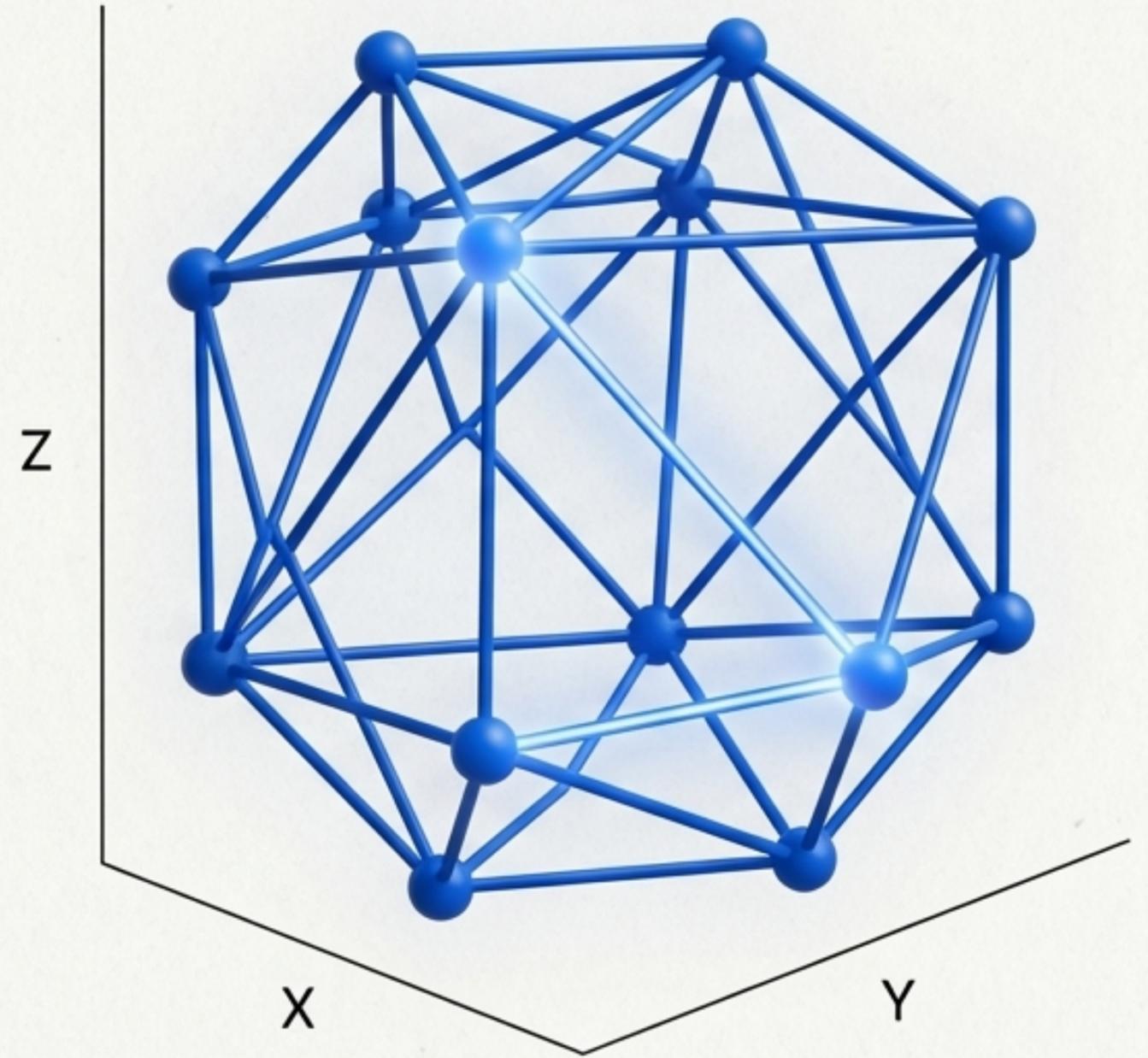
The distance between adjacent vertices is constant, confirming a uniform edge length across the entire structure.

Edge Length Verification

$$\|(1, 1, 0) - (1, 0, 1)\|^2 = 0^2 + 1^2 + (-1)^2 = 2$$

**Thus, all edges satisfy:**

$$l = \sqrt{2}$$



# The Blueprint Realized: Topological Closure

Vertices (V): **12**  
Edges (E): **24**  
Faces (F): **14**



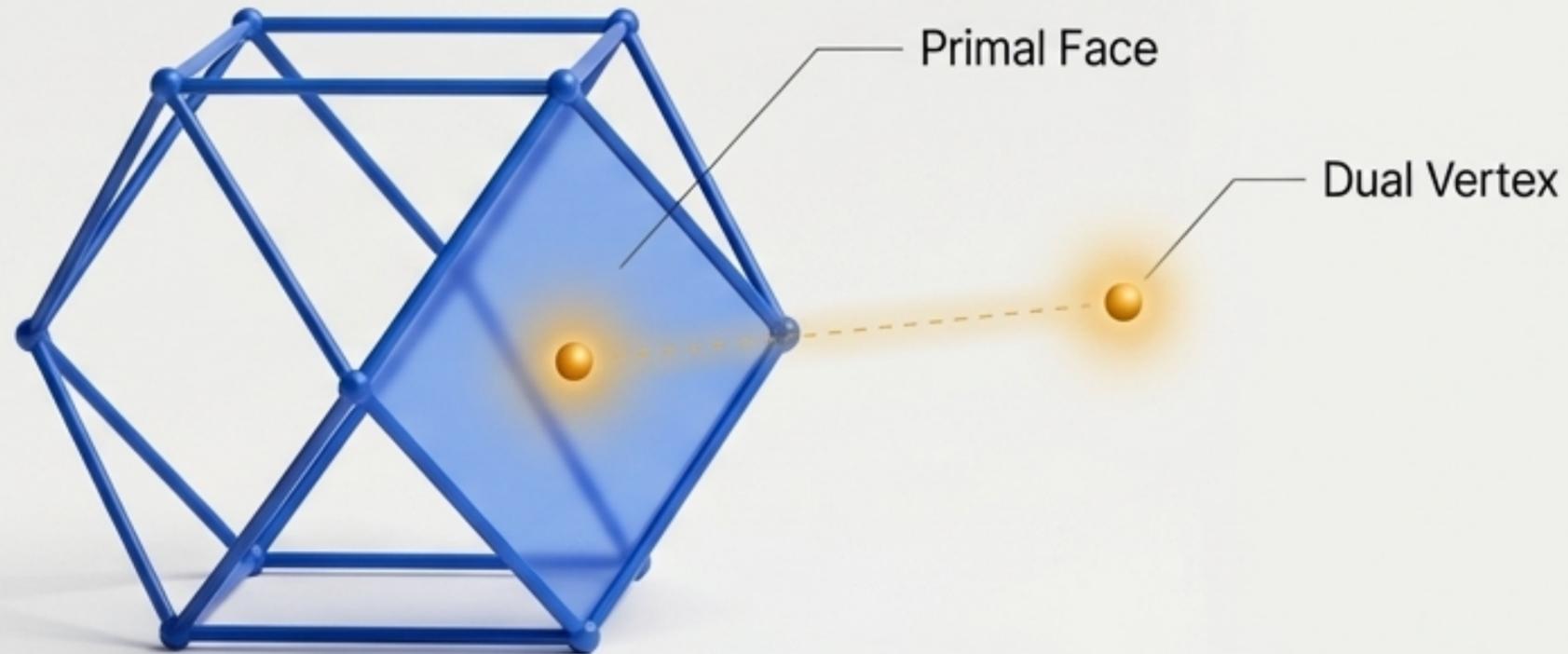
The 14 faces are composed of two distinct regular polygons, all sharing the same edge length:

- **8** equilateral triangles
- **6** squares

# The Echo: Constructing the Dual Solid

Every polyhedron has a dual, a corresponding solid where the roles of faces and vertices are interchanged. We can generate this dual directly from the geometric data of our primal solid.

## The Dual Construction Principle

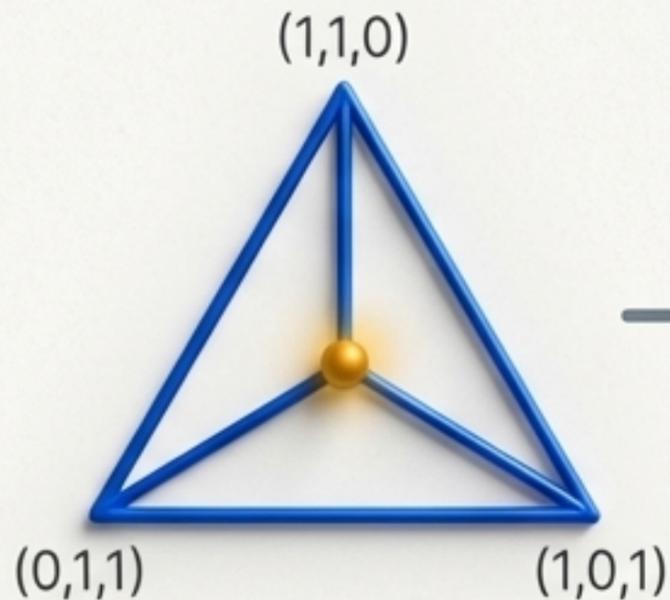


The vertices of the dual polyhedron are located at the centroids of the faces of the primal solid. Crucially, no scaling or normalization is applied. The form is derived in its pure state.

# Deriving Dual Vertices from Face Centroids

We calculate the centroids for each of the 14 faces of the cuboctahedron to find the vertices of its dual.

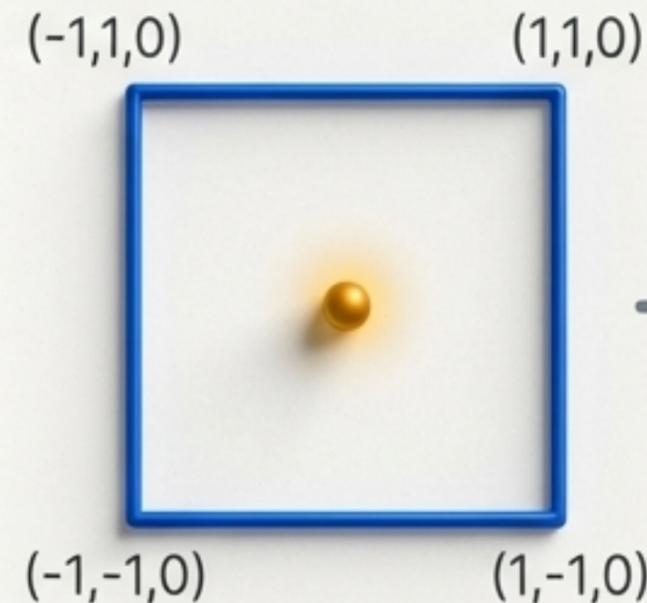
From the 8 triangle faces:



The centroids yield vertices of the form:  
 **$(\pm 1, \pm 1, \pm 1)$**

*(This generates 8 vertices)*

From the 6 square faces:



The centroids yield vertices of the form:  
 **$(\pm 2, 0, 0), (0, \pm 2, 0), (0, 0, \pm 2)$**

*(This generates 6 vertices)*

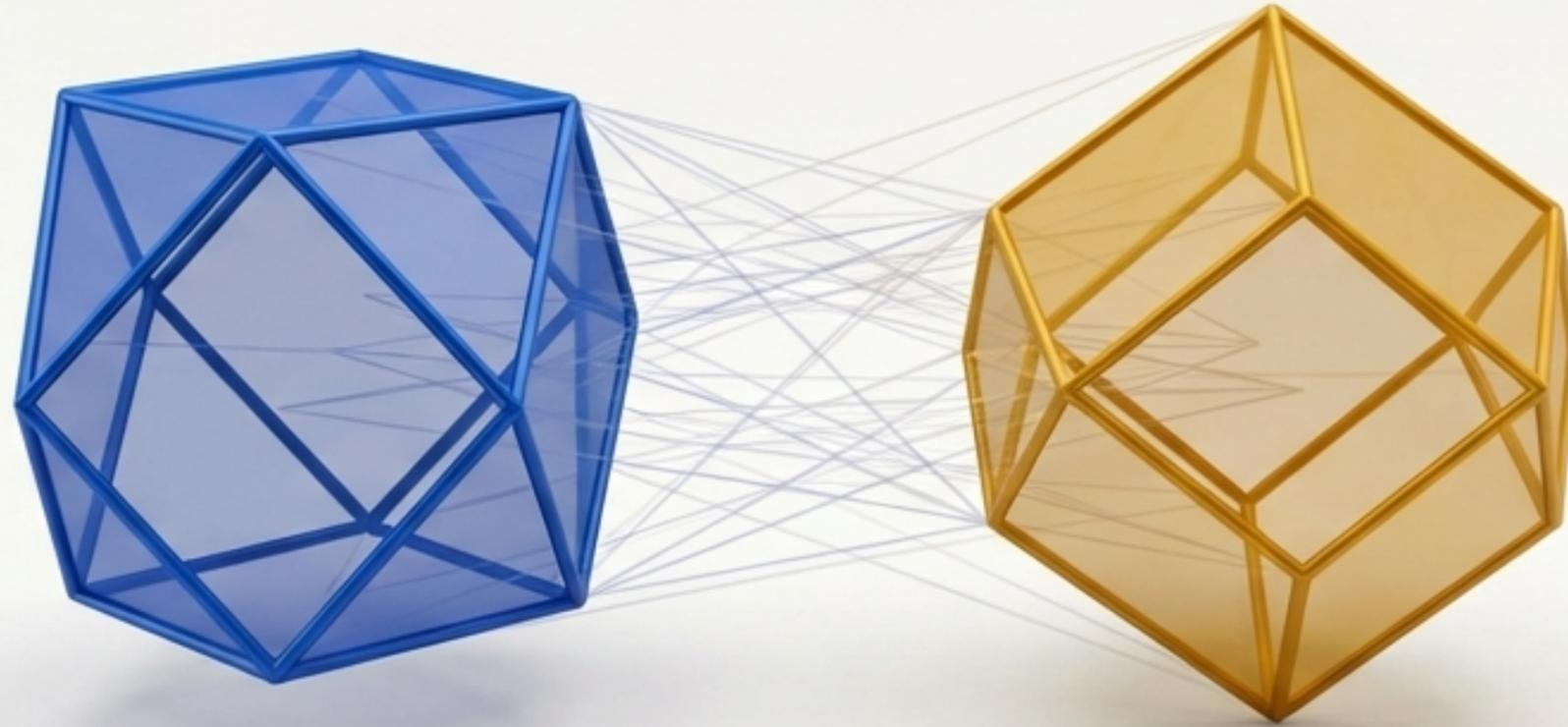
# The Echo Realized: The Rhombic Dodecahedron

Vertices (V): 14  
Edges (E): 24  
Faces (F): 12



The combined set of 14 dual vertices forms a new solid. All edges connect vertices at equal distances, producing 12 congruent rhombic faces. The shared edge count ( $E=24$ ) confirms the correct duality.

# A Duality in Form: Blueprint and Echo



## Primal: Cuboctahedron

$V = 12$

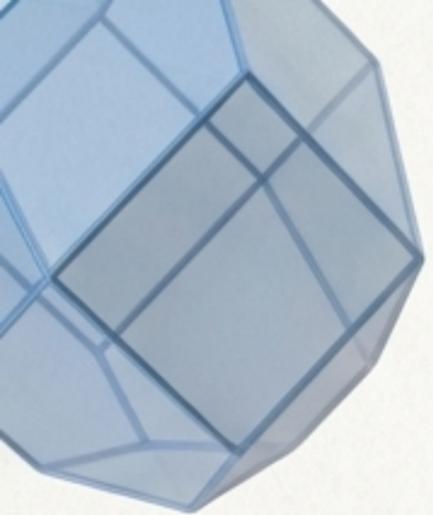
$F = 14$  (8 Triangles, 6 Squares)

## Dual: Rhombicuboctahedron

$V = 14$

$F = 12$  (12 Rhombi)

The 12 vertices of the Cuboctahedron correspond to the 12 faces of the Rhombic Dodecahedron.  
The 14 faces of the Cuboctahedron correspond to the 14 vertices of the Rhombic Dodecahedron.



# The Generative System Summarized

Solid	Vertices	Factor Set	Generative Rule
<b>Cuboctahedron</b>	12	{1}	Permuted $(\pm 1, \pm 1, 0)$
<b>Rhombic Dodecahedron</b>	14	{1, 2}	Derived from Dual Centroids