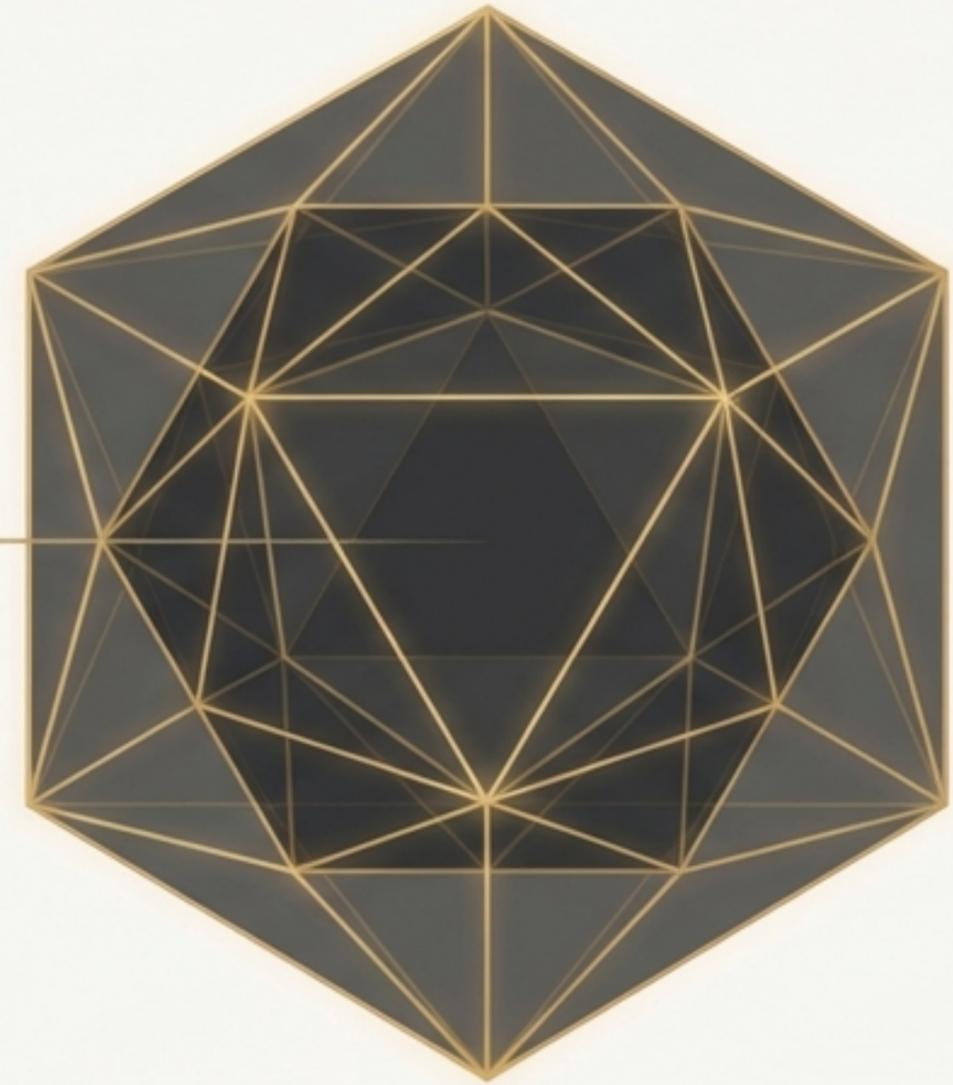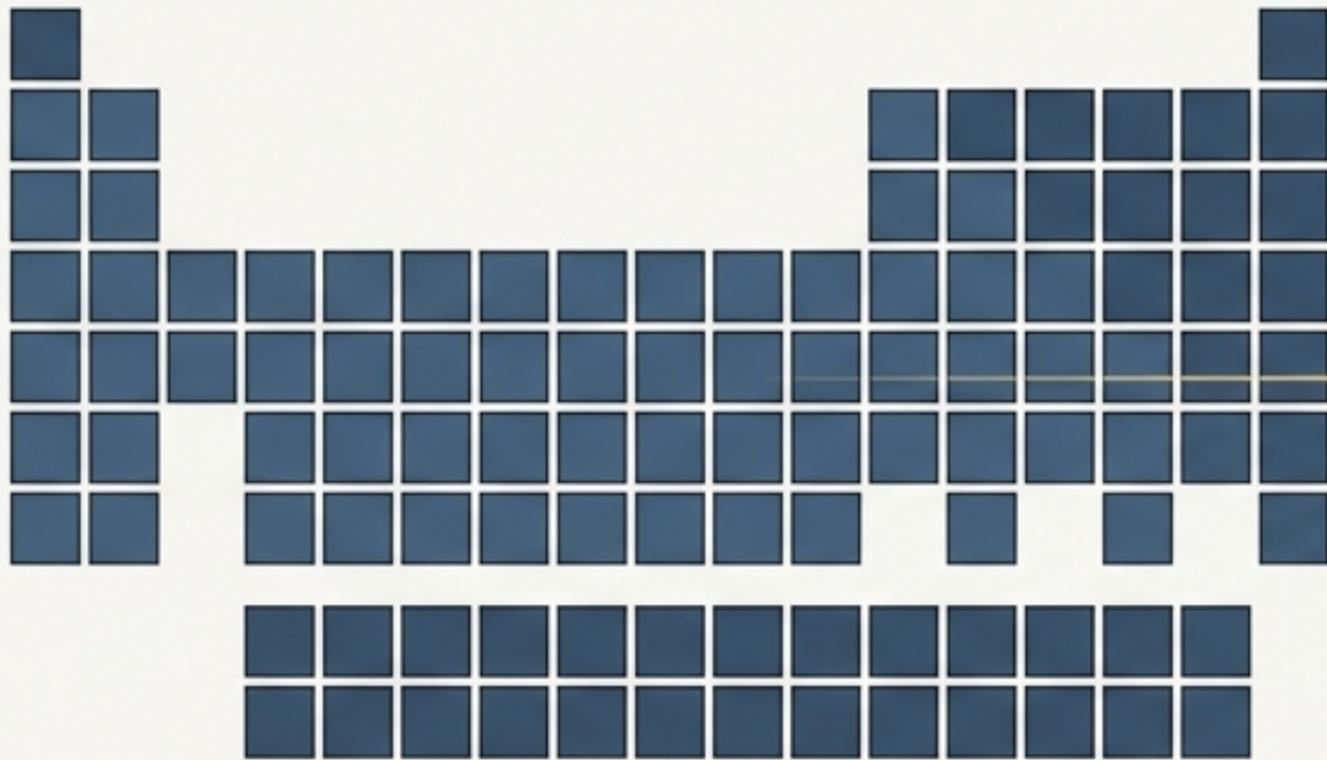# What if the blueprint for the elements was hidden in plain sight?


Proposing a unified geometric framework for atomic structure, derived from first principles.

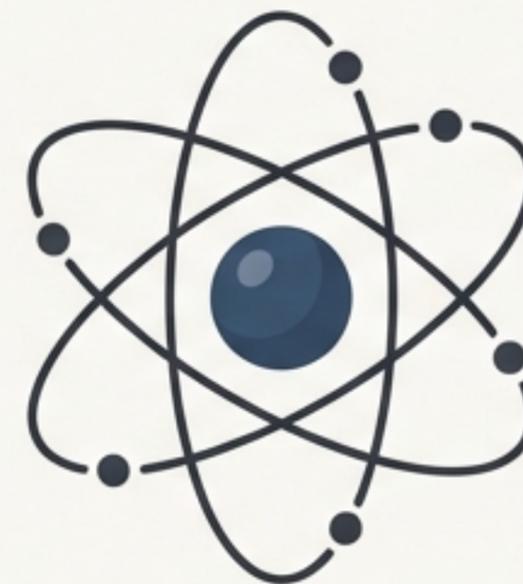# The Core Hypothesis: Atoms as Grant Polytopes

The fundamental correspondence is that each element maps to a position in a Pythagorean triple cascade. Electron shells map to the Nine Harmonic Means, and chemical behavior emerges from the resulting polyhedral geometry.
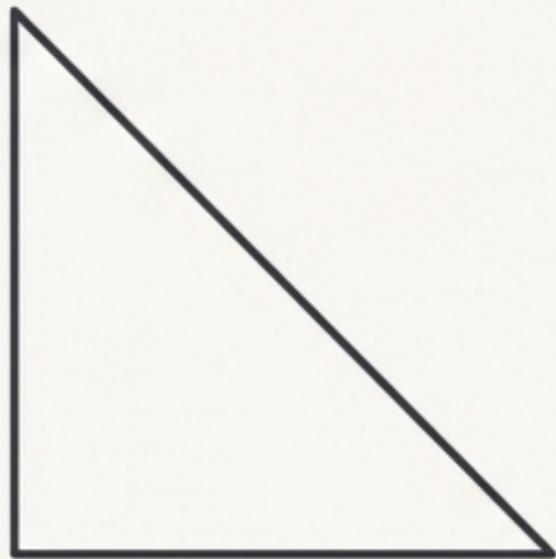
Pythagorean Triple.
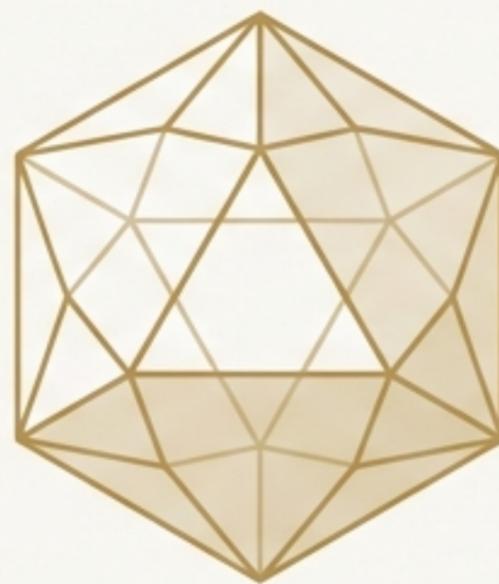
Grant Polytope Projection.

Element & Chemical Properties.

The Generative Engine: The '+1 Pythagorean Cascade'

The framework is built upon the consecutive-leg Pythagorean family, an orthogonal factor chain where c – b = 1. This cascade provides the fundamental geometric scaffold for atomic structure.

$$(a_n, b_n, c_n) = (2n+1, 2n(n+1), 2n(n+1)+1)$$

n=1 → **(3, 4, 5)**
↓
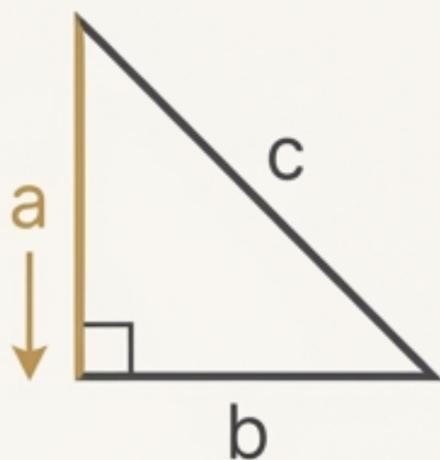n=2 → **(5, 12, 13)**
↓
n=3 → **(7, 24, 25)**
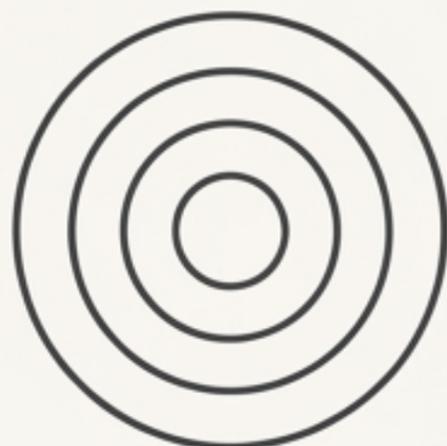↓
n=4 → **(9, 40, 41)**
↓

...

The Geometric Derivation Rules

Rule 1: Seed Triple:
$a_n = 2n+1$
(Proton Seed / Period Start).

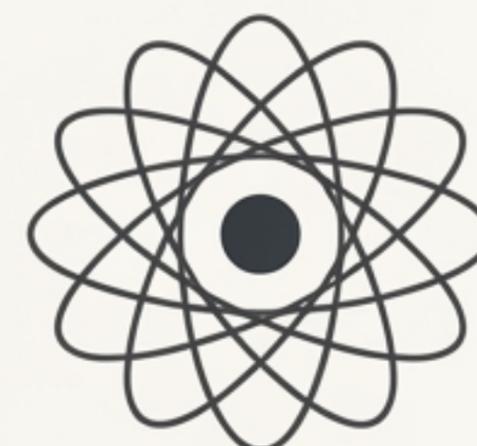**Rule 2: Electron Shells:**
$b_n = 2n(n+1)$
(Basis for 9 Harmonic Mean Shells).

Rule 3: Orthogonal Chains:
s/p-blocks map to the Main +1 Chain. d/f-blocks map to Parallel Chains.
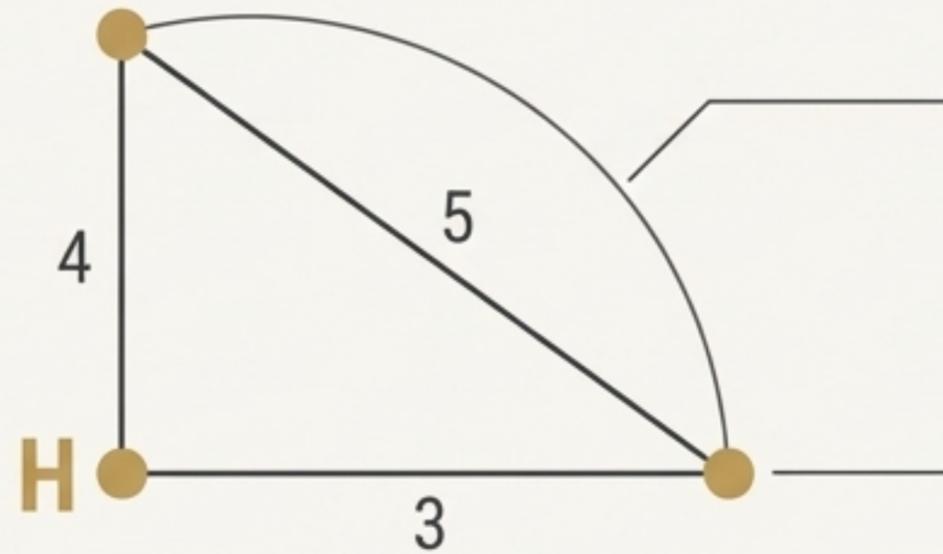
Rule 4: Geometric Bonds:
Bonds are defined as Inter-shell Edges. Hybridization arises from Face Mixing.

Rule 5: Stability
A maximum of 9 shells dictates stability. The next orthogonal chain predicts new elements.
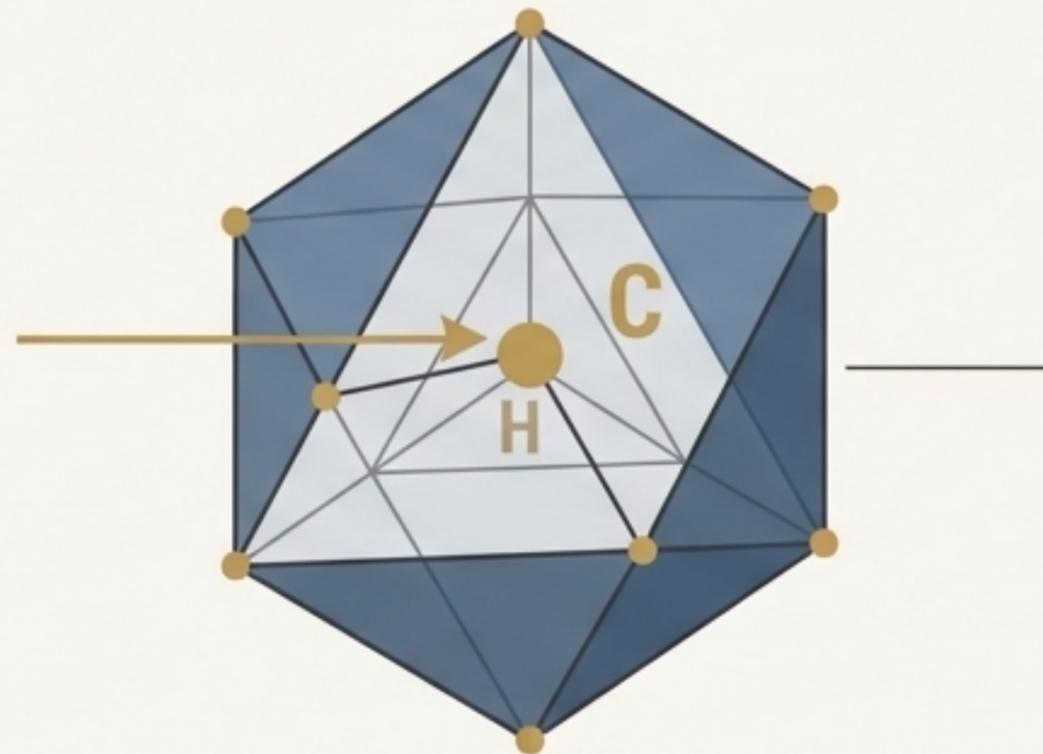
Period 1 & 2: The Primordial Triple (3, 4, 5)

Helium (Z=2): Two vertices complete the triangular core, a geometrically closed shell.

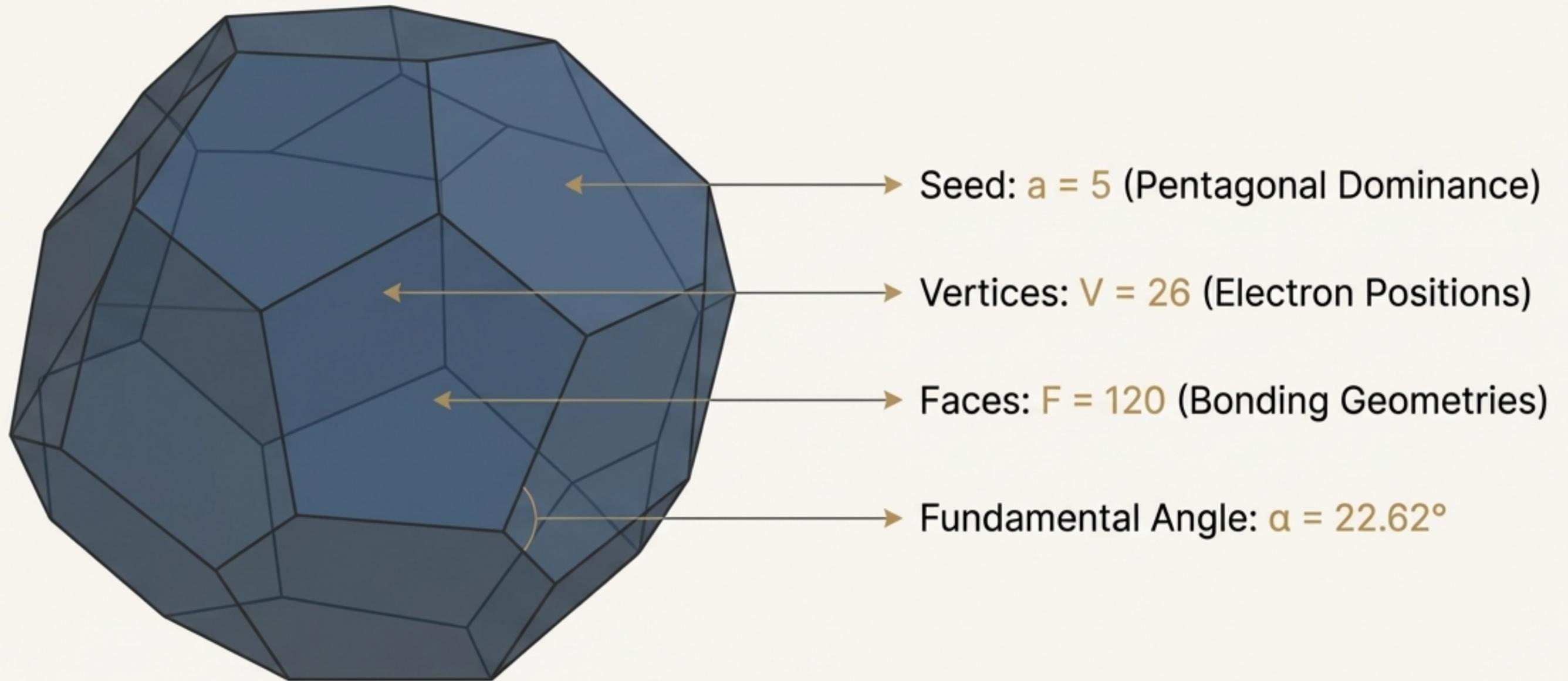**Hydrogen (Z=1):** A single vertex on the innermost shell.

Carbon (Z=6): Centrality enables tetrahedral hybridization.

Lithium-Neon (Z=3-10): The octet rule emerges naturally from the 8-vertex extensions of the primordial geometry.

Periods 3 & 4: The Alphahedron Regime (5, 12, 13)

For n=2, the cascade generates a pentagonal-dominant geometry. This 'Alphahedron' accommodates elements Sodium to Krypton (Z=11-36) and its structure is foundational to organic chemistry.

Seed: $a = 5$ (Pentagonal Dominance)

Vertices: $V = 26$ (Electron Positions)

Faces: $F = 120$ (Bonding Geometries)

Fundamental Angle: $\alpha = 22.62°$

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The Nine Harmonic Means as Electron Shells

DHM (1s)
DM (2s)
DQM (2p)
HM (3s)
GM (3p)
AM (3d / 4s)
QM (4p)
LBM (4d / 5s)
LBM (4f / 5p+)
LGM (4f / 5p+)
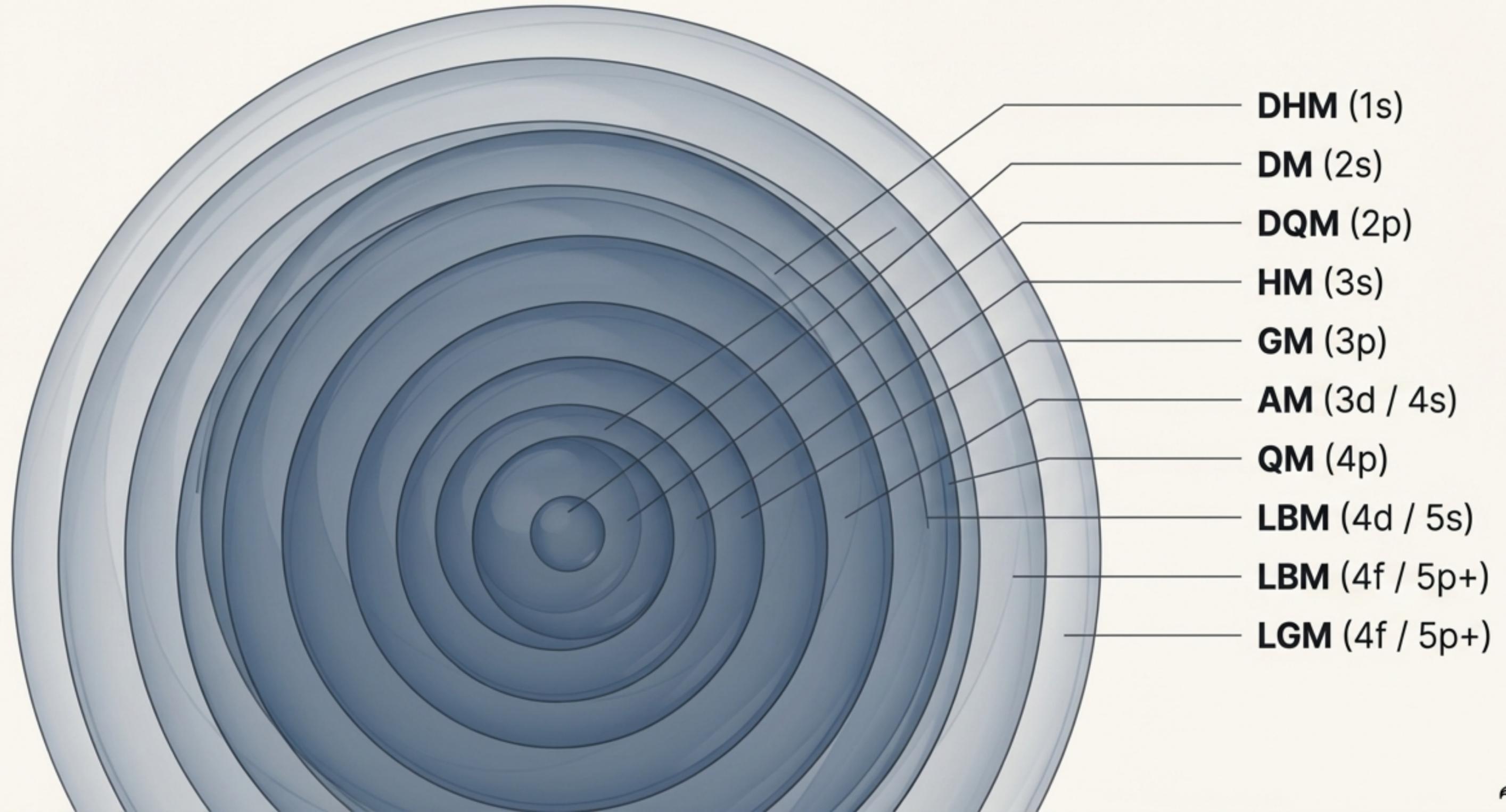
Chemical Bonding and Hybridization Emerge from Geometry
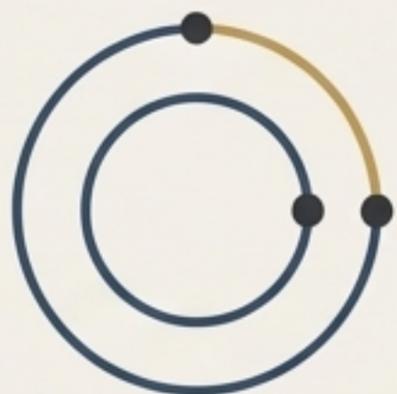
σ bond: Intra-shell edge.
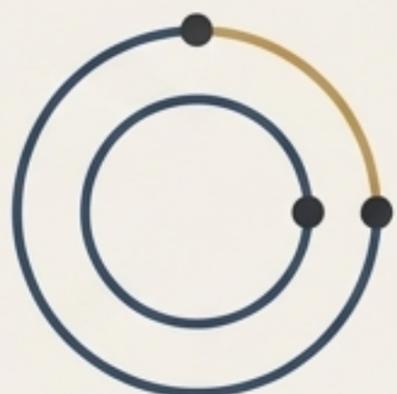
π bond: Inter-shell edge.
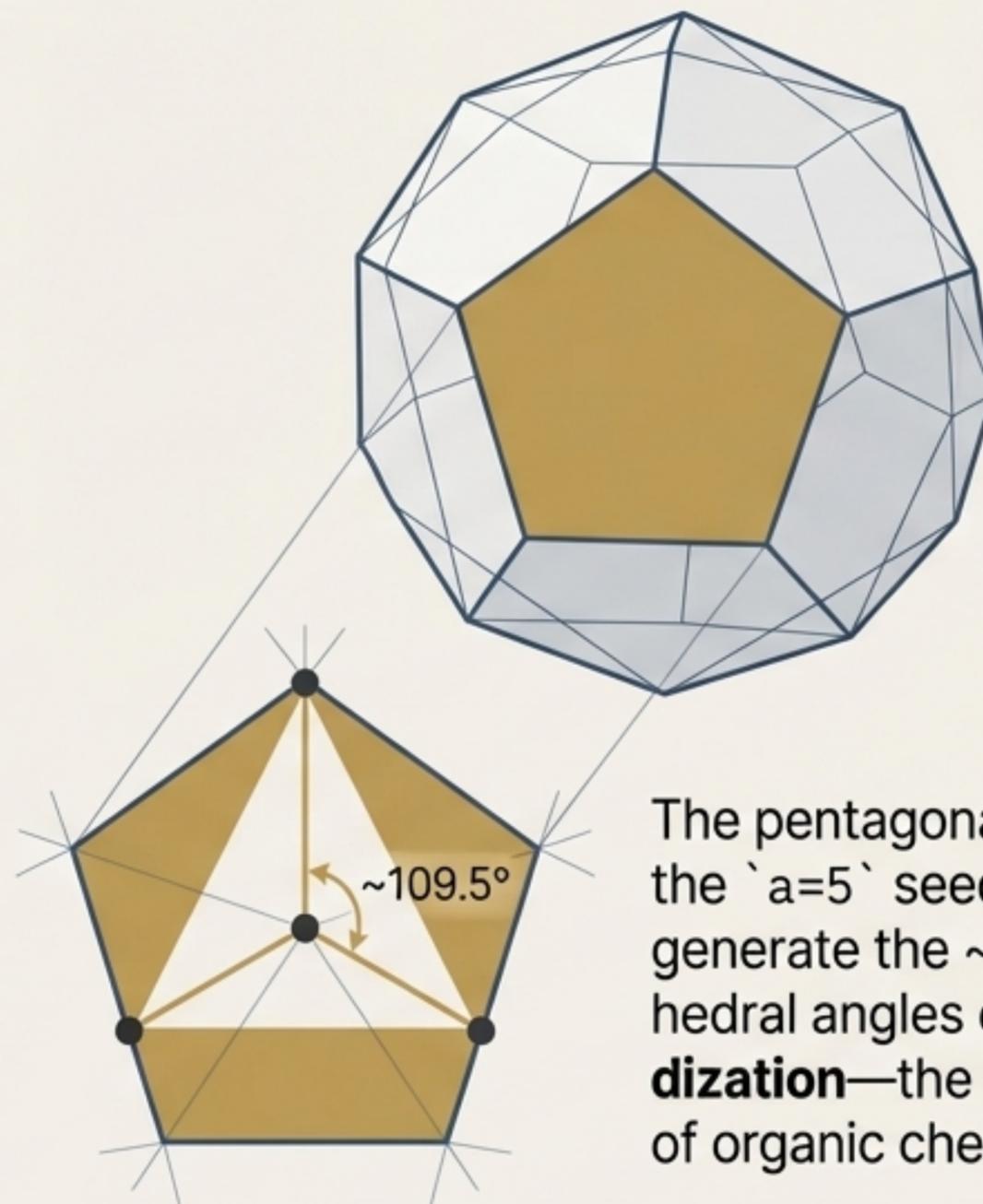
Ionic bond: Shell-to-shell transfer.

~109.5°

The pentagonal faces from the `a=5` seed naturally generate the ~109.5° tetrahedral angles of **sp³ hybridization**—the cornerstone of organic chemistry.

A Geometric Limit to the Elements

The model proposes a maximum of nine stable harmonic mean shells. The periodic table's stability limit corresponds to the complete filling of this geometric structure.

"Theorem 6.1: Maximum stable atomic number corresponds to complete filling of all 9 harmonic shells: $Z_{max} \approx V_{total}$."
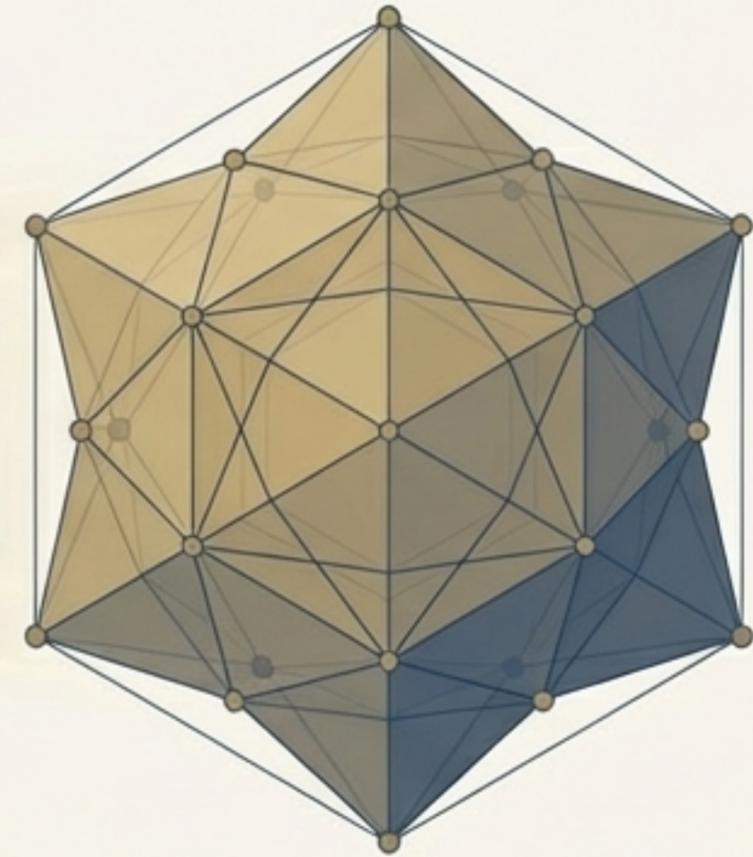

Oganesson (Z=118)
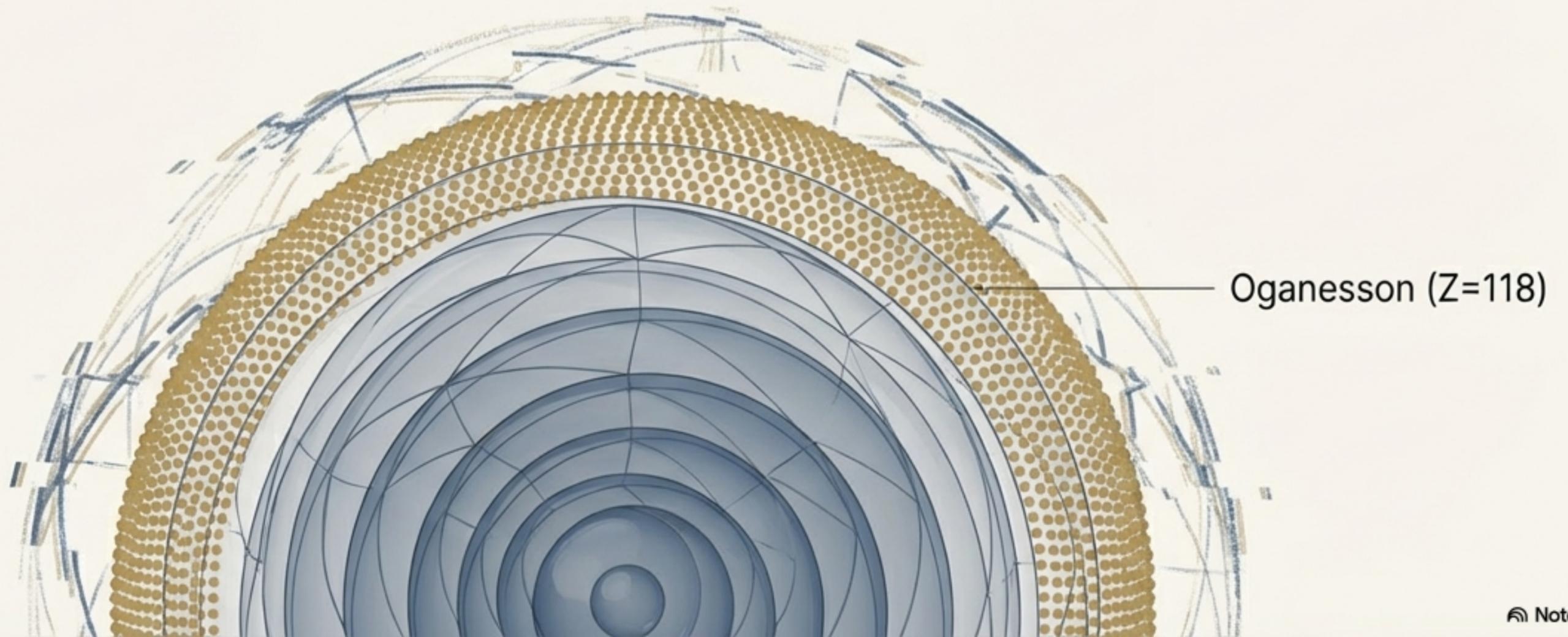
Predicting the Island of Stability

Superheavy elements ($Z > 118$) are predicted to exist where stability is conferred by the geometric closure of a new shell. This arises from the *next orthogonal chain* in the Pythagorean cascade.
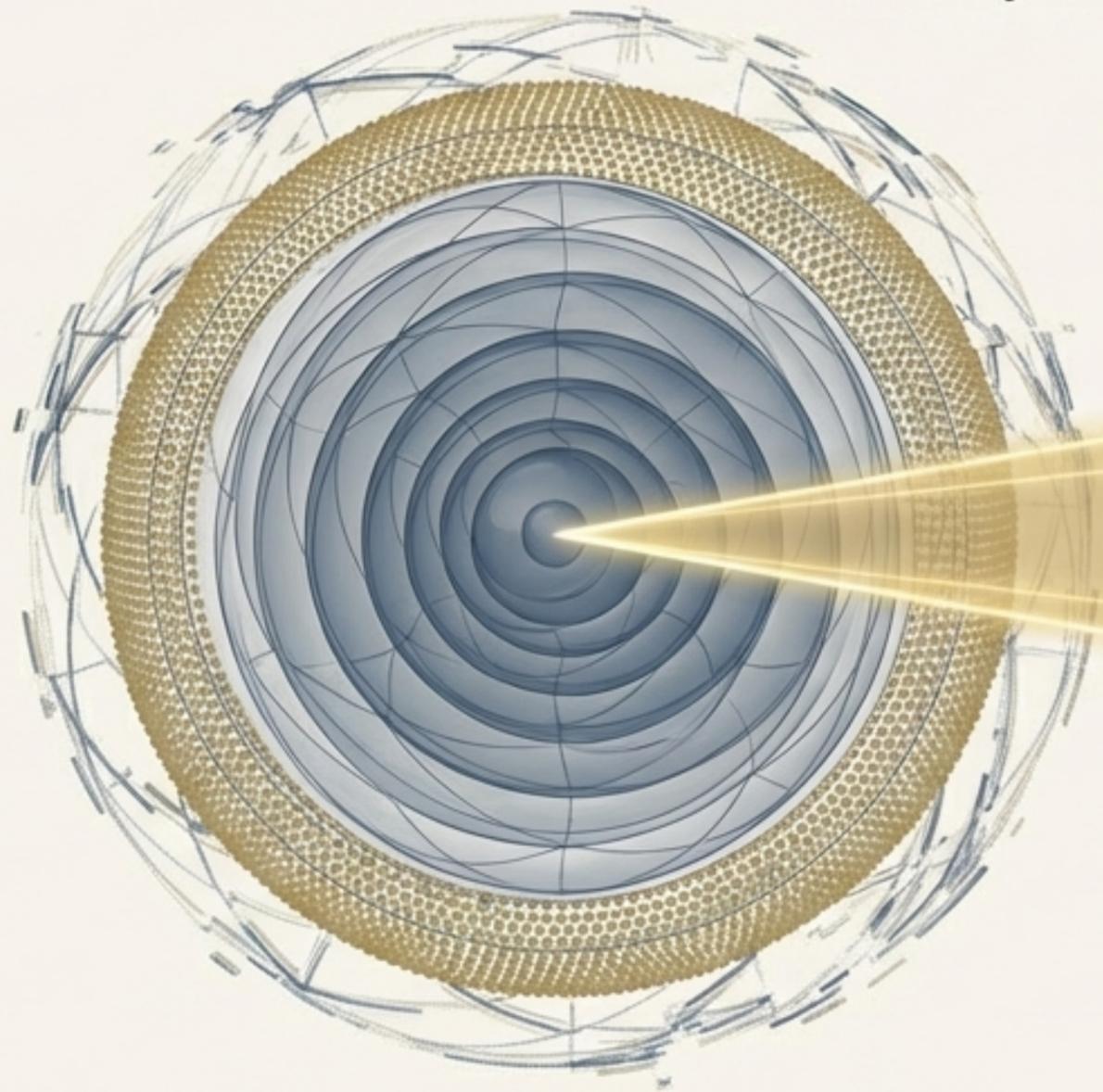
Next Orthogonal Cascade

Predicted Island of Stability

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The Geometry of the Next Elements

The next step in the cascade (n=5) generates the triple (11, 60, 61). The geometry of the resulting polytope provides specific predictions for the properties of the next stable superheavy elements.

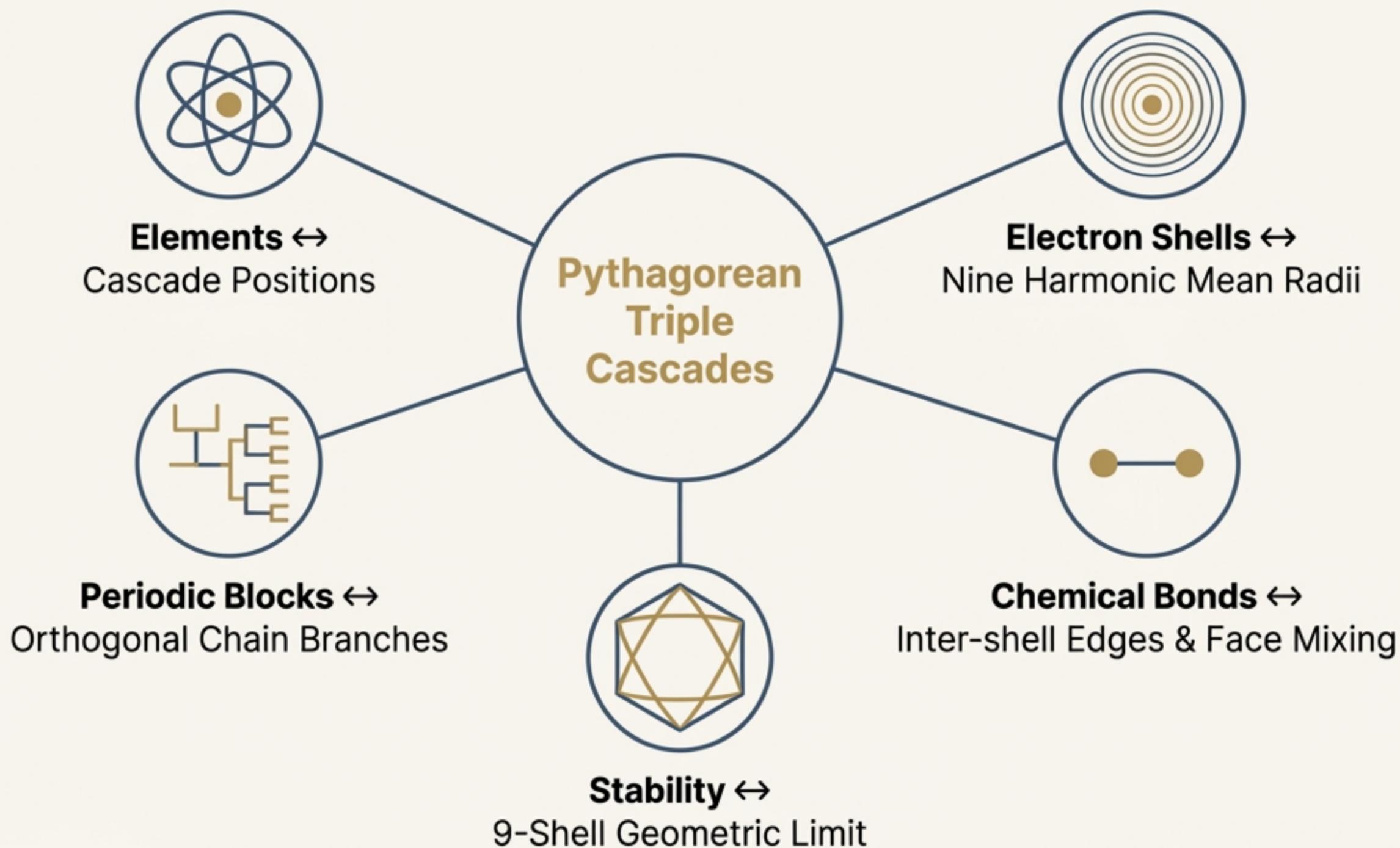## Generating Triple

(11, 60, 61)
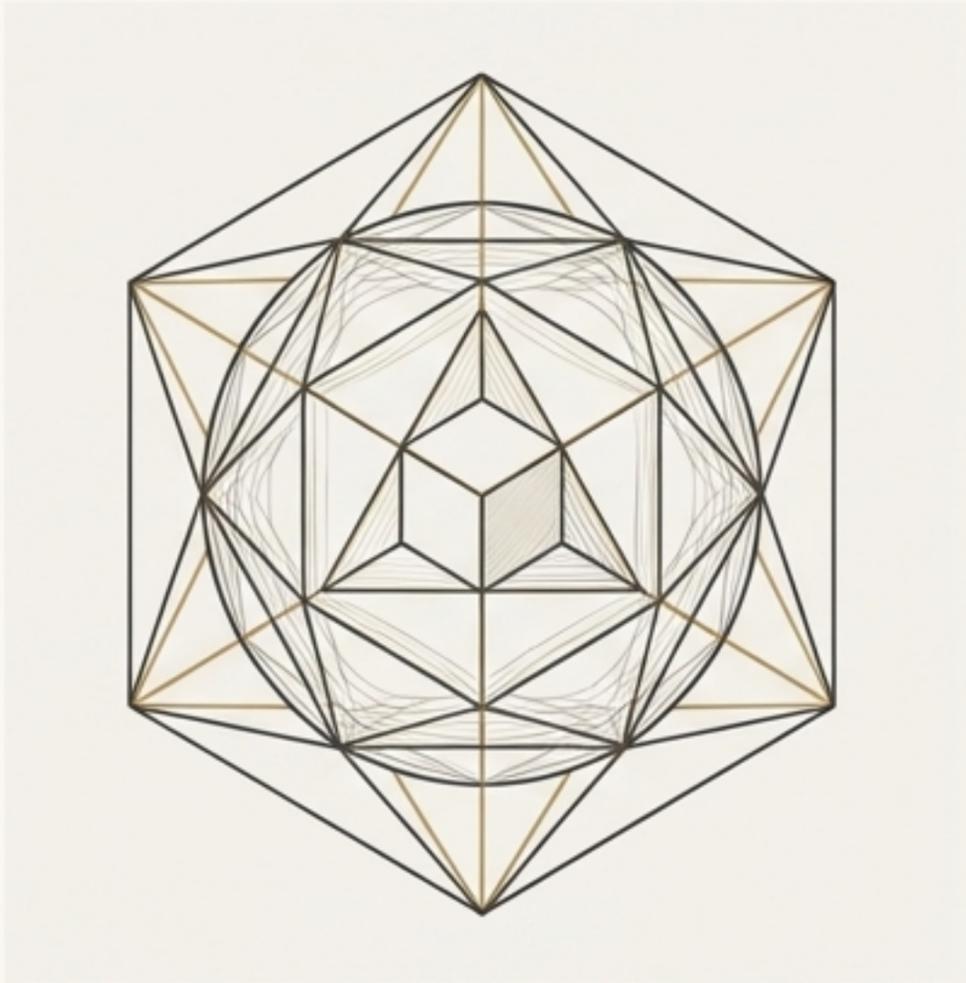
Predicted Geometric Properties

– Vertices (electrons): $V = 122$

– Edges (bonds): $E = 3600$

– Faces (geometries): $F = 3480$

This framework suggests a new region of nuclear stability centered around $Z \approx 120$–126.

From a Single Principle: A Unified Geometric Chemistry

Pythagorean Triple Cascades

Elements ↔
Cascade Positions

Electron Shells ↔
Nine Harmonic Mean Radii

Periodic Blocks ↔
Orthogonal Chain Branches

Chemical Bonds ↔
Inter-shell Edges & Face Mixing

Stability ↔
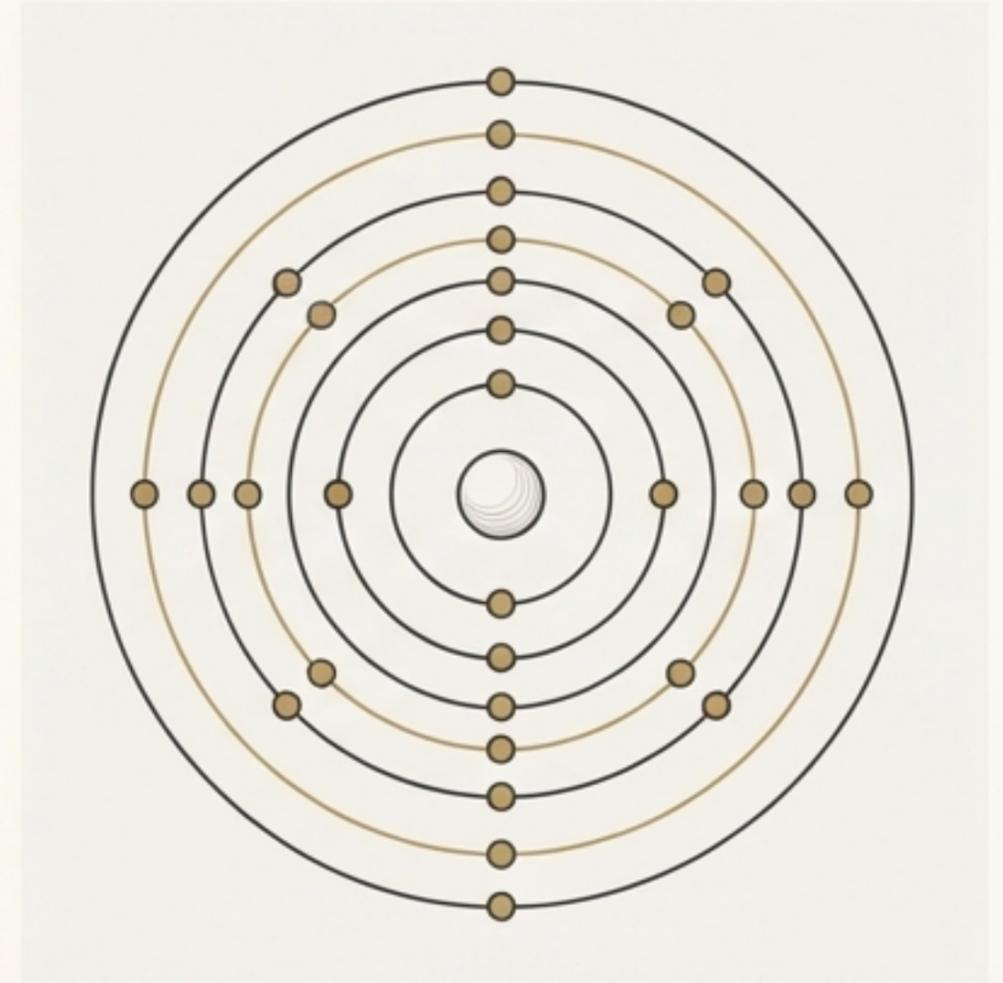9-Shell Geometric Limit

A Modern Synthesis in a Lineage of Ideas

Parallels historical nested polyhedral models.
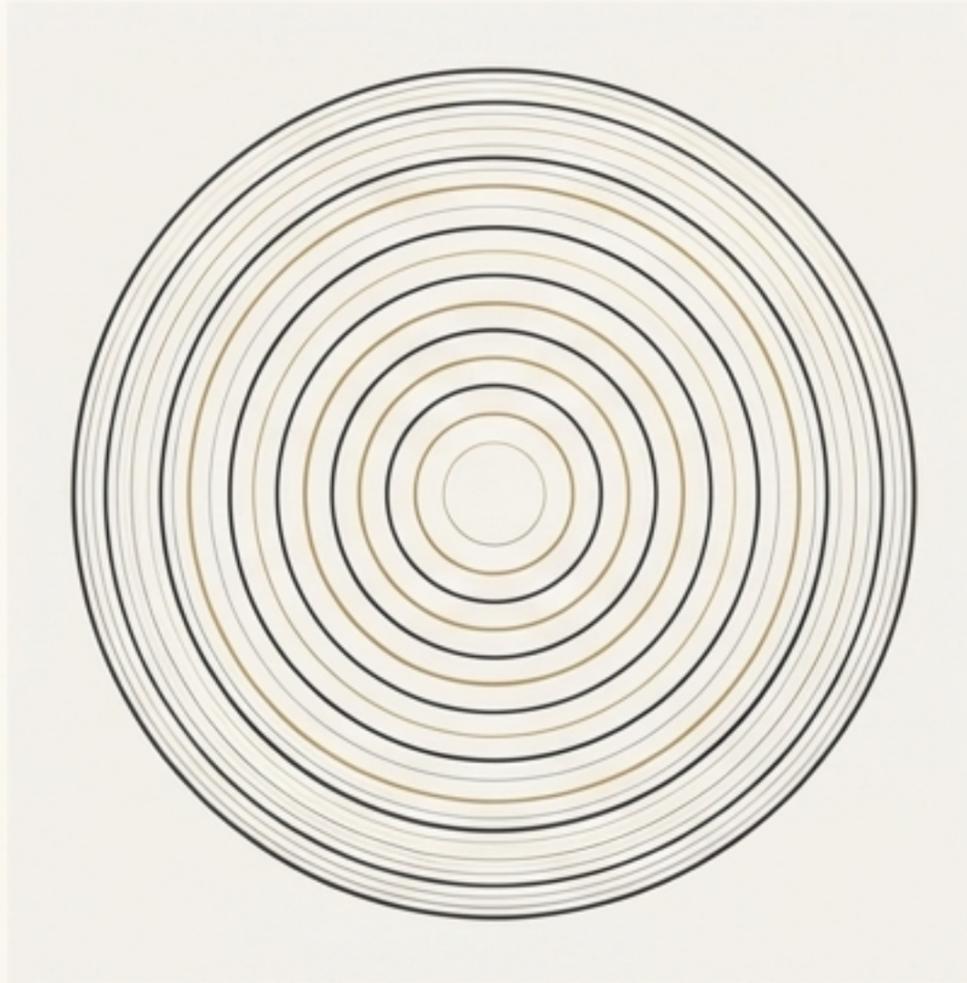
Unifies geometry with quantum principles.

Extends concentric sphere models with harmonic structure.

An Invitation to a New Frontier

This geometric framework offers a new lens through which to explore the fundamental nature of matter. The next steps are to test, refine, and extend its predictive power.

Key Future Directions:
- **Validate**: Compare predicted shell radii to experimental atomic radii.
- **Derive**: Calculate spectroscopic signatures from geometric shell transitions.
- **Extend**: Apply the mapping to nuclear structure and molecular geometries.
- **Guide**: Use predicted magic numbers ($Z \approx 120–126$) to guide superheavy element synthesis.

Source & Further Reading

Title: Mapping Periodic Elements to Grant Polytopes: A Geometric Framework for Atomic Structure via Pythagorean Triple Cascades

Author: Robert Edward Grant

Publication Date: December 2025

Contact: robert@robertedwardgrant.com

Link: arXiv:2512.XXXXX [physics.atom-ph]

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