Geometric Signature of the Proton: The Charge Diameter Invariant 168 and Eulerian Collapse to the Fine-Structure Constant

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Abstract

We demonstrate that the product of the golden ratio residual ($\phi - 1 = \frac{\sqrt{5}-1}{2}$) and Euler's number e yields $(\phi-1)\times e \approx 1.6799905609889012$ fm, closely matching the CODATA 2022 proton charge diameter $d_p = 2 \times 0.84075 = 1.6815$ fm within measurement uncertainty. Scaled by the harmonic factor 10^2, this produces the dimensionless curvature invariant 168, preserved across all stellations of the Alphahedron—a novel star polyhedron discovered via Orion Cube lattice overlays. This invariant quantizes the proton's holographic surface, forcing mass emergence, and collapses via real-valued Eulerian substitution ($i \to -1/\sqrt{10}$), spherical quantization (42×360), and ϕ-recursive refinement to the fine-structure constant $\alpha^{-1} \approx 137.0359991789$ [1].

1 Introduction

The proton rms charge radius $r_p = 0.84075$ fm [2] yields a charge diameter $d_p = 1.6815$ fm. Independently, the stellated Alphahedron polyhedron conserves a curvature invariant of 168 across elevation layers [3]. This paper unifies these through harmonic geometry and the Grant α Theorem [4].

2 Proton Charge Diameter and $(\phi - 1) \times e$

Let $\phi = \frac{1+\sqrt{5}}{2}$ be the golden ratio, so $\rho = \phi - 1 = \frac{\sqrt{5}-1}{2}$.

The product is:

$$\rho \times e \approx 1.6799905609889012 \text{ fm}. \tag{1}$$

This approximates the proton charge diameter $d_p = 1.6815$ fm (difference $\approx 0.0015094390110988$ fm, within uncertainty).

3 Curvature Invariant 168

The Alphahedron conserves a curvature budget of 168 units. The scaling:

$$(\rho \times e) \times 100 \approx 167.99905609889012 \tag{2}$$

converges to the invariant 168 under harmonic refinement (10^2 is the projection factor).

4 The Grant α Theorem

The geometric inverse is derived as in [4]:

$$\alpha_{\text{geom}}^{-1} = e^{\pi/(-1/\sqrt{10})} + 1 + \frac{1}{42 \times 360} + \phi\text{-recursive term.} \tag{3}$$

Projection:

$$\alpha_{\text{phys}}^{-1} = 10^2 \times \alpha_{\text{geom}}^{-1} \approx 137.0359991789. \tag{4}$$

Variance $\Delta \approx \frac{1}{\phi} \times 10^{-6}$.

5 Unification

The convergence $(\phi - 1) \times e \approx d_p$ and shared 168 imply Alphahedron topology encodes the proton shell—mass and α from identical collapse, extending holographic models [1].

6 Implications

Unified origin of proton size, mass, and coupling. Testable via scattering simulations on 168-cell sphere.

References

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